

Energy-based Approach for Dissipative Structural Glass System in Seismic Regions

Manuel Santarsiero, Kosmas Moupagitsoglou

Eckersley O'Callaghan, London, manuel@eocengineers.com, kosmas@eocengineers.com

Current design codes and standards provide limited indications for advanced analysis and earthquake engineering of structural glass applications in seismic regions. This work provides an energy-based approach for efficient design and structural performance evaluation of structural glass systems in seismic regions. The analytical formulation of the energy-based approach for dissipative non-linear structural glass systems is firstly presented. A practical application is then described by means of analytical and numerical studies. The results show that the combination of appropriate structural design with advanced non-linear analysis allows the achievement of highly efficient design and satisfactory performances comparable to the ones of other common structural systems.

Keywords: Earthquake engineering, Dissipative structural design, Structural glass, Energy-based approach

1. Introduction

The use of glass in structural applications has been significantly increased in the past years. However, glass structures are still a relatively new domain and the design of structural components made of glass (and other innovative materials) is often not covered by standards and codes. The latter, indeed, usually provide indication only for standard materials and simple constructions and do not provide indications for advanced analysis and earthquake engineering of structural glass applications in seismic regions. Because of the above, two main issues arise. Firstly, structural glass components are often designed based on very conservative and unrealistic assumptions. This leads to (i) inaccurate assessment of performances and safety level (ii) inefficient use of material in structural components and (iii) onerous requirements to substructures, which lead to expensive overdesign. The second issue is the limited information available on the appropriate approaches and the structural benefit of advanced design of efficient (i.e. dissipative) structural glass components and systems in seismic regions. There is indeed the need to establish analytical methods and design approaches to engineer structural (glass) systems able to provide satisfactory structural dynamic non-linear performance in the event of an earthquake. Based on the above motivations, this initial work provides an analytical energy-based approach for efficient design and realistic structural performance evaluation of structural glass systems in seismic regions. Firstly, the structural performance and analysis methods for structural systems in locations with seismic activity is briefly recalled. Then, an energy-based approach for dissipative non-linear structural glass systems is described. Finally, a simple application is presented by means of analytical and numerical studies.

2. Performance of structural systems in seismic region

Structural systems located in seismic region are characterized by a significant probability to experience earthquake events and the related ground motion phenomena. In the field of structural engineering, acceleration, velocity and displacement experienced over time at ground surface during ground motion events are of main interest. These values recorded over time are usually called time-history diagrams. Time-history plots are then analyzed with Fourier transformation to evaluate the dynamic characteristics of the ground motion of a certain seismic event over a range of period and frequency (as acceleration, velocity and displacement). These are then plotted as a function of the period or frequency. Such diagrams, often called spectrum plots, are useful to identify the dynamic characteristic of a ground motion event and often represent the main inputs provided by codes and standards for seismic analysis of structural systems.

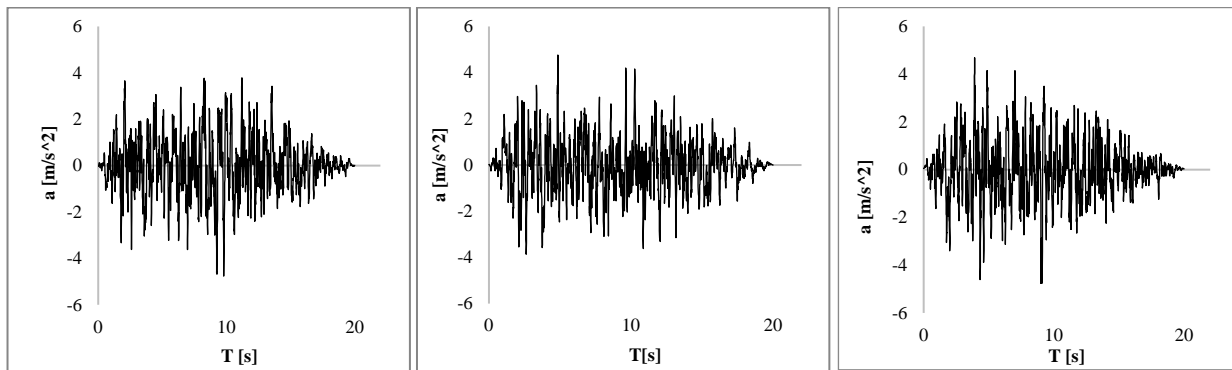


Fig 1. Example of simulated time-history graphs of ground motion seismic events

Structural systems designed against earthquake action must be characterized by adequate safety and serviceability performance capacity in the case of a seismic event. Among others, one of the main requirements is that the probability of total collapse of a structural system should be limited to allowable maximum probability (defined by national codes as a function of the use of the construction and the class of consequence). This implies that at ultimate limit state, although yielding, plastic deformation and extensive structural damages are allowed, collapse should not occur (or, more precisely, the probability of collapse should be limited to allowable value defined by the relevant codes and standards, in line with fully probabilistic structural design approaches).

The effects of a certain seismic event on a structural system depend on its dynamic performance and how it is related with the dynamic characteristic of the excitation under consideration. Different approaches can be used to evaluate the structural performance of dynamic systems during seismic events. These approaches are characterized by different level of complexity. The appropriate approach to be used depends not only on the level of accuracy required, but also on the type of structure under consideration (e.g. in terms of geometry, materials, systems, location, ground geotechnical characteristics, etc.).

Computationally, the most complex approach is the non-linear step-by-step dynamic analysis with geometry and material non-linearity implementation. This approach requires the evaluation of the dynamic non-linear response of the structures by the simulation of several time-histograms that simulate relevant ground motion events in multiple directions and in. A main complexity of such an approach, in addition to being complex and expensive, is the appropriate choice of the relevant time-history diagrams, subject not discussed here for the sake of brevity. This approach allows the evaluation of the energy dissipation that the structure will exhibit during seismic events. The dissipation can be related to materials and components yielding, plastic deformation, hysteresis cycles and other damages phenomena that dissipate energy during ground motion events.

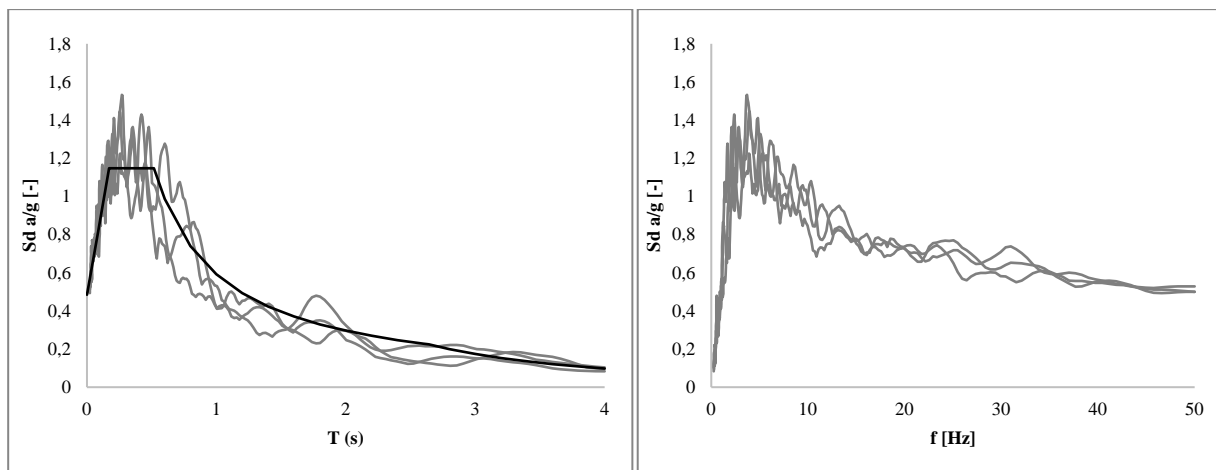


Fig 2. Spectrum plots of time history ground motions versus (left) time and (right) frequency

A less computationally expensive non-linear approach is the so-called push-over analysis. It consists of a non-linear static step-by-step analysis, that allows the derivation of the capacity non-linear curve (along one relevant direction). In this approach, the maximum drift that a structural system can exhibit is computed with the capacity curve, accounting for all dissipative phenomena that could occur before the ultimate collapse. The ratio between the ultimate displacement and the displacement at the limit of the elastic region provides an indication of the system ductility and the capacity of energy dissipation along the selected direction (see the following section for more details). In addition to providing general information on the ultimate non-linear behaviour of the structure, it is used in comparison with

the capacity demand curve of relevant seismic events. This can only be carried out along one direction at time, thus the choice of the appropriate direction is a quite critical aspect, especially for irregular building.

Spectrum modal analysis and equivalent static analysis are less complex, yet approximate approaches, widely suggested by seismic code and standards. These approaches use elastic analysis, hence they would not be able to account for any energy dissipation and damage occurring during a seismic event. This means that they cannot directly account for the actual ultimate capacity and safety of the structural system against a certain seismic event. To account for the above-mentioned non-linear and dissipative phenomena, standards and codes define the so-called design spectrums to be used in the linear approximate equivalent analysis. The design spectrums are obtained from the elastic spectrum, which is then divided by a reduction factor, also called “*behaviour factor*” (q) in the Eurocode framework, or “*response factor*” in the American standard framework. In the following behaviour factor name will be used for clarity. The behaviour factor permits the consideration of the dissipative capacity of the structural system in the linear approximate analyses. This allows the use of elastic analyses (either modal or static equivalent) yet to consider the non-linear dynamic characteristics and dissipative capacity of the structural system by means of a ‘reduced’ design spectrum. The behaviour factor therefore depends on several aspects that influence the dissipative capacity of the structural system, such as the type of material, components, structural systems, geometry of the structure, etc. Codes and standards usually provide reference values of behaviour factor for common materials and simple structural systems. For non-standard material or complex structural systems, the actual behaviour factor needs instead to be directly evaluated.

The following section presents an energy-based approach and the related analytical equations for the estimation of the behaviour factor for a given structural system.

3. Energy-based approach for dissipative structural glass system under dynamic action

Different approaches can be used to evaluate the behaviour factor of a structural system to be used in the approximate equivalent linear analysis. Among others, like displacement-based approach and acceleration-based approach, the so-called energy-based approach is widely used by international codes and standards. It provides satisfactory results for structural systems with the main natural frequency within the range of the maximum amplitude of the relevant seismic spectrum, which covers most of typical building constructions. The main concept of this approach is to equalize the energy of an indefinitely elastic system to the energy dissipated by the actual non-linear system during the seismic event. More specifically, to equalize the energy of the elastic system to the energy dissipated up to the ultimate displacement that the structure can exhibit under dynamic ground motion (Xu in Fig. 3).

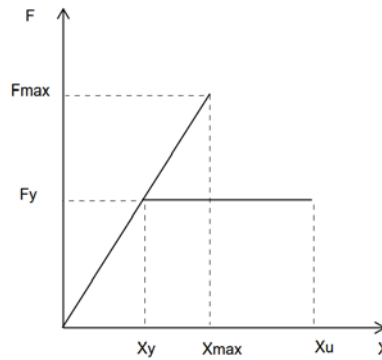


Fig 3. Schematic representation of the energy-based approach

Figure 3 shows a schematic representation of the two systems that are used in this section, an elastic one and an elastic-plastic one. Using the notation provided in Fig. 3 the below equation can be derived:

$$\frac{F_{max} \cdot X_{max}}{2} \cong F_y \cdot X_u - \frac{F_y \cdot X_y}{2}$$

Then

$$\frac{F_{max}}{F_y} = \frac{2X_u - X_y}{X_{max}}$$

Defining

$$q = \frac{F_{max}}{F_y} ; \mu = \frac{X_u}{X_y}$$

Given

$$\frac{F_{max}}{X_{max}} = \frac{F_y}{X_y} \rightarrow \frac{F_{max}}{F_y} = \frac{X_{max}}{X_y} \rightarrow X_{max} = q \cdot X_y$$

Then

$$q = \frac{F_{max}}{F_y} = \frac{2X_u - X_y}{q \cdot X_y} \rightarrow q^2 = 2 \frac{X_u}{X_y} - 1 = 2\mu - 1$$

Thus

$$q = \sqrt{2\mu - 1}$$

The above equations can be used to estimate the behaviour factor of a given system and thus, to derive the approximate design spectrum to be used in equivalent linear analysis (such as modal dynamic analysis or equivalent static analysis). It should be noted, that several theoretical assumptions and practical simplifications are made, which are not described here in detail for the sake of brevity and because outside the scope of this work. For example, the above equations consider the simple non-linear behaviour typology depicted in Fig. 3, i.e. initial elastic response followed by a perfectly plastic response after the first yielding. For more complex behaviour, the energy dissipation of the non-linear systems should be computed as the integral of the graphs up to the ultimate nonlinear response.

From the equations above it can be observed that the behaviour factor is proportional to the ductility of the system (μ), i.e. ratio between the ultimate displacement capacity and the displacement correspondent to the initial deviation from linear elastic behaviour. This is in line with the expectation, as the more ductile is the structure the more energy the system is capable to dissipate during a seismic event.

The above equations allow, for any type of material and structures based on the property and structural performance capacity the system, the computation of the reduction factor to be used to estimate the design seismic spectrum for approximated elastic analyses (see figure 4).

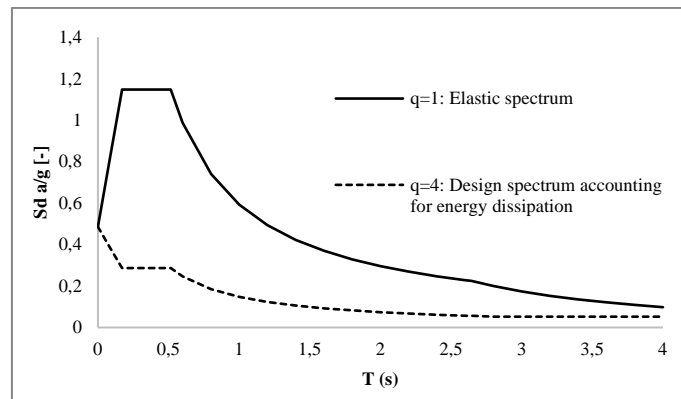


Fig 4. Elastic and design spectrum accounting for behaviour factor (example for Italy - L' Aquila, with return period 475 years).

This approach is of interest for new materials and for non-standard structural systems, for which design standards do not yet provide indication on the appropriate behaviour factor. This approach therefore allows the enhancement of the actual performance assessment of structural glass system under seismic event and to increase the efficiency of their structural designs. This is not only applied with regards to the ultimate resistance, but more importantly in seismic regions, to the dissipative capacity and ductility of the structural system. Enhanced assessment of the structural performance and safety level can indeed be performed by means of the above presented approach together with the computation of the actual non-linear behaviour and dissipative capacity of the structural system under consideration.

In that regard, the following section provides a brief description of a typical system used in structural glass applications, together with a case study where ductility and behaviour factors are computed using analytical and numerical approaches.

4. Evaluation of dissipative capacity with energy approach - case study

Based on what it has been described so far, it is a common practice glass structures to be designed using the elastic earthquake response spectra. By following this approach, we assume that the structure cannot dissipate any seismic energy and therefore it needs to resist the full seismic action (behaviour factor $q=1.0$). Given the brittle nature of glass, and the fact that only recently it has started to be used as a load bearing structural material, this conservative approach has been in general acceptable, although resulting in extremely conservative structural assessment and inefficient use of material. In the following case study, we are presenting a novel way to approach the design of glass structures located in seismic regions. More specifically, the use of structural systems that resist seismic loads in the nonlinear

range, which permits design with seismic forces smaller than those corresponding to a linear elastic response (behaviour factor $q > 1.0$).

4.1. Case study

The structural system of the case study (Fig. 5) is a portal frame that consists of two, 6m high glass columns and a glass beam that spans 8m. All the elements are made of laminated glass and their depth and width is equal to 600mm and 66mm respectively (5x12mm heat-strengthened glass laminated with 1.52mm ionoplast interlayer). The connections between the columns and the beam are pinned. This allows a more efficient accommodation of the movements that might occur at support structure. In addition, the lateral stability of the system is achieved through push-pull moment connections at the base the columns.

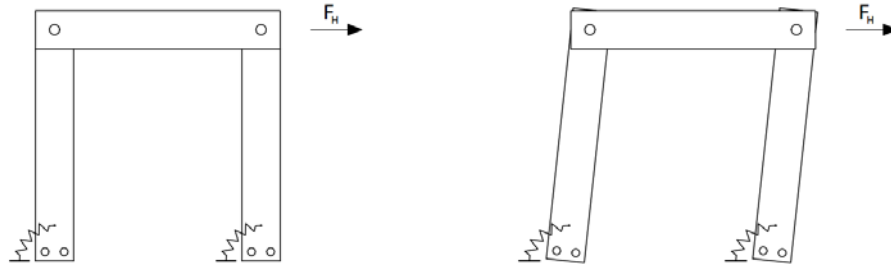


Fig 5. Schematically representation of the structural glass system for the case study (left) undeformed (right) under lateral displacement

More in detail, two stainless steel pins passing through two holes at the bottom of each glass column are here considered. The bolts are then connected to four mild steel angles that are in turn bolted to the base structure (Fig. 6). As described in the following sections, these connections and their structural design will play an important role in the ultimate dissipative capacity of the structural system and thus on the possibility to reduce the equivalent seismic action via the behaviour factor.

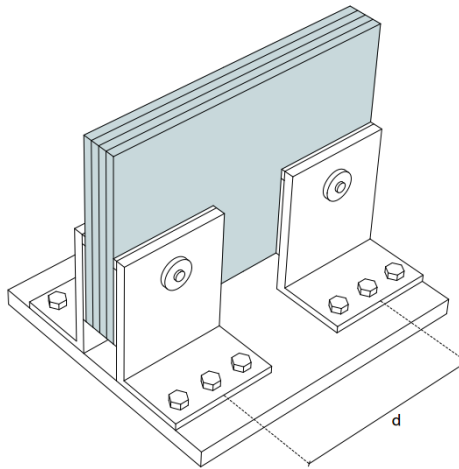


Fig 6. Schematic representation of the base connection of the case study

4.2. Analytical study

To estimate the behaviour factor as described in the previous section it is necessary to compute the ultimate displacement capacity of the structural system under consideration. In the current case the ultimate horizontal movement of the glass frame is mainly linked to the deformation capacity of the base connection. The in-plane elastic deformation is neglected in this study for the sake of brevity. The lateral drift of the frame (Δ) can be calculated as a function of the deformation and rotation of the bottom connections of the columns, i.e. as a function of the vertical deformation of the steel angles (δ) (see Fig. 7).

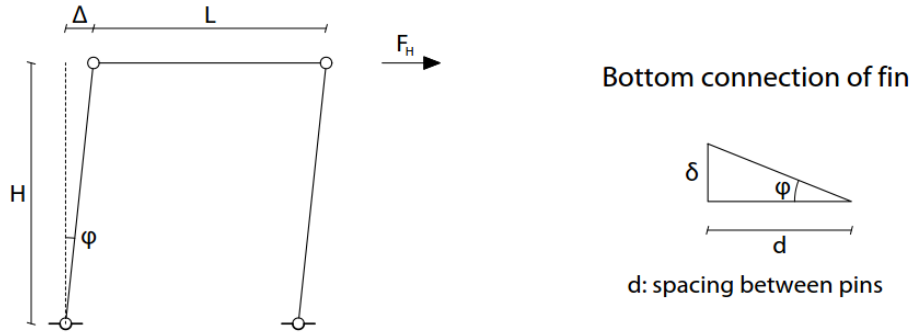


Fig 7. Scheme of the structural system

$$\tan \varphi = \frac{\Delta}{H} = \frac{\delta}{d} \rightarrow \Delta(\delta) = \frac{H}{d} \times \delta$$

The vertical deformation of the angles at the first yielding can be calculated analytically as follows:

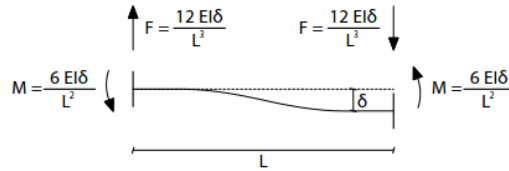


Fig 8. Analytical relationship between imposed displacement and bending moment

$$M = \frac{6 \times E \times I \times \delta_y}{L^2} \rightarrow \frac{b \times t^2}{6} \times f_y = \frac{6 \times E \times I \times \delta_y}{L^2} \rightarrow \delta_y = \frac{f_y \times L^2}{3 \times E \times t}$$

For 100mm distance between the ‘fixed’ supports, 15mm thickness and S235 mild steel the above equation can be written as follows:

$$\delta_y = \frac{f_y \times L^2}{3 \times E \times t} \rightarrow \delta_y = \frac{235MPa \times (100mm)^2}{3 \times 210000MPa \times 15mm} \rightarrow \delta_y = 0.248mm$$

Similarly, we can calculate the vertical deformation of the angles just before failure (ultimate limit state) (Fig. 9):

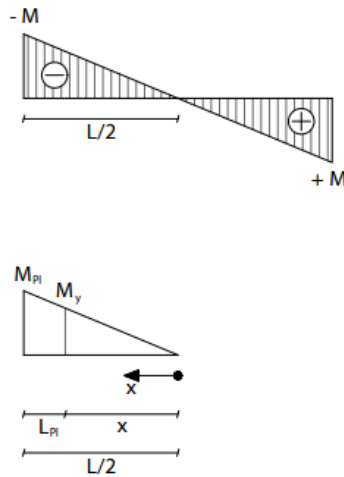


Fig 9. Determination of the location at first yielding

Using the following formulas, we can estimate the length of the theoretical plastic hinge extension:

$$\frac{x}{M_y} = \frac{L/2}{M_{pl}} \rightarrow x = \frac{M_y}{M_{pl}} \times \frac{L}{2}$$

$$L_{pl} = \frac{L}{2} - x \rightarrow L_{pl} = \frac{L}{2} \times \left(1 - \frac{M_y}{M_{pl}}\right)$$

Assuming the actual rotation point is approximatively located at the centre of the plastic hinge, the vertical deformation of the angle at the ultimate state, considering the curvature distribution (see Fig. 11), would be:

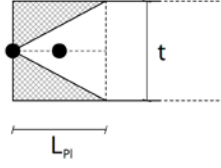


Fig 10. Scheme of the plastic deformation at the plastic hinge

$$\delta_u = \delta_y + \theta_{pl} \times (L - L_{pl})$$

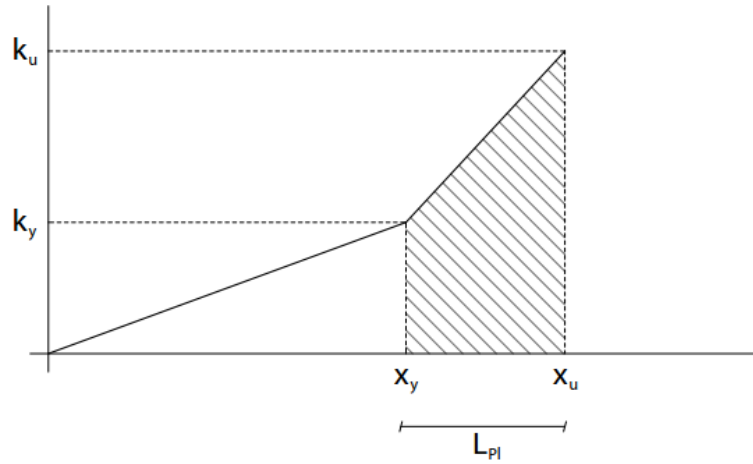


Fig 11. Diagram of the non-linear curvature in the structural element

$$d\theta = k(x) \times dx$$

$$\theta_{pl} = \int_{x_y}^{x_u} k(x) \times dx = (x_u - x_y) \times k_y + \frac{(x_u - x_y) \times (k_u - k_y)}{2} \rightarrow$$

$$\theta_{pl} = L_{pl} \times \frac{\varepsilon_y}{(t/2)} + \frac{L_{pl} \times (\varepsilon_u/t/2 - \varepsilon_y/t/2)}{2} \rightarrow$$

$$\theta_{pl} = \frac{L_{pl} \times (\varepsilon_y + \varepsilon_u)}{t}$$

For the steel angle we have already described:

$$L_{pl} = \frac{L}{2} \times \left(1 - \frac{M_y}{M_{pl}}\right) \rightarrow L_{pl} = \frac{100mm}{2} \times \left(1 - \frac{1}{1.5}\right) \rightarrow L_{pl} = \frac{100}{6} mm$$

For a 15mm thick S235 mild steel plate:

$$\varepsilon_y = \frac{235MPa}{210000MPa} \times 100\% = 0.112\% \text{ \& } \varepsilon_u = 25\%$$

Using the above equations:

$$\delta_u = \delta_y + \theta_{pl} \times (L - L_{pl}) = \delta_y + \frac{L_{pl} \times (\varepsilon_y + \varepsilon_u)}{t} \times (L - L_{pl}) \rightarrow$$

$$\delta_u = 0.248mm + 23.251mm = 23.499mm$$

With the above information, the ductility and behaviour factor can be therefore analytically estimated:

$$\mu = \frac{\delta_u}{\delta_y} = \frac{23.499\text{mm}}{0.248\text{mm}} = 94.75$$

$$q = \sqrt{2 \times \mu - 1} = \sqrt{2 \times 94.75 - 1} = 13.72$$

4.3. Numerical study

In order to verify the analytical study, a FEA plane-strain model is created using the software Strand7. The numerical model is used to derive the vertical deflection of the steel angles at first yielding (δ_y) and the ultimate vertical deflection just before failure (δ_u). The ductility (μ) and the behaviour factor (q) can then be calculated using the previously described equations. As for the stress-strain curve, it is assumed an elastic modulus of steel after yielding equal to $E/10000=21$ MPa (in line with EN 1993-1-5, Annex C, clause C.6). The following images show outputs of the numerical analysis. Fig. 12 shows the yield index at the first yielding with displacement equal to 0.50mm and Fig. 13 shows the distribution of the plastic strain at the ultimate limit state ($\epsilon=25\%$) with displacement equal to 40.25mm. Finally, Fig. 14 shows the yield index at the ultimate limit state.

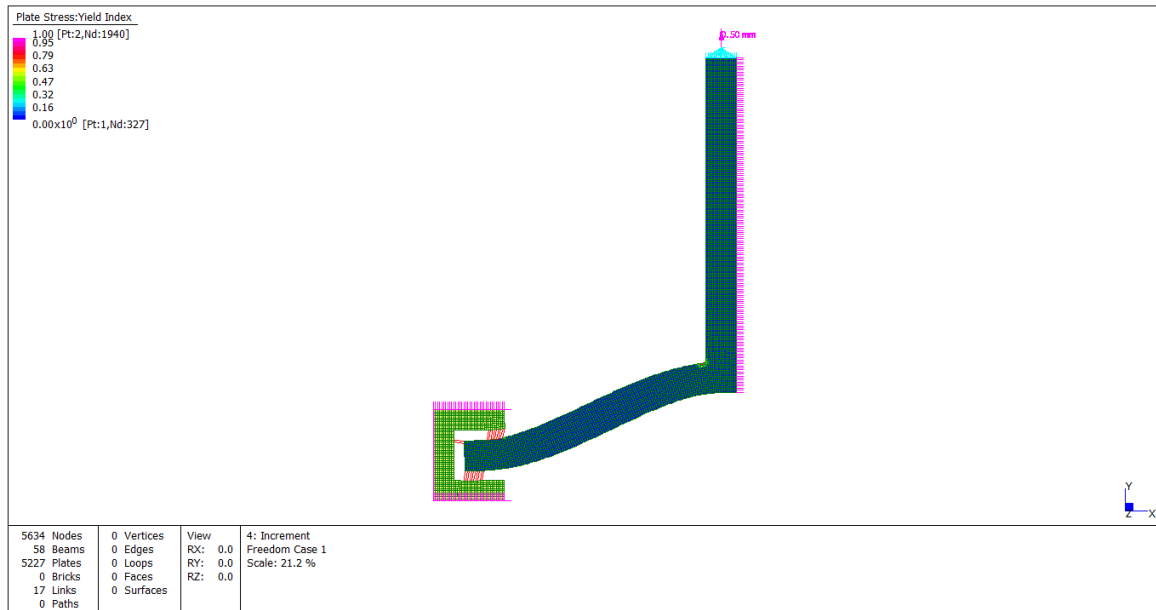


Fig 12. Output of the numerical simulation at first yielding – yield index

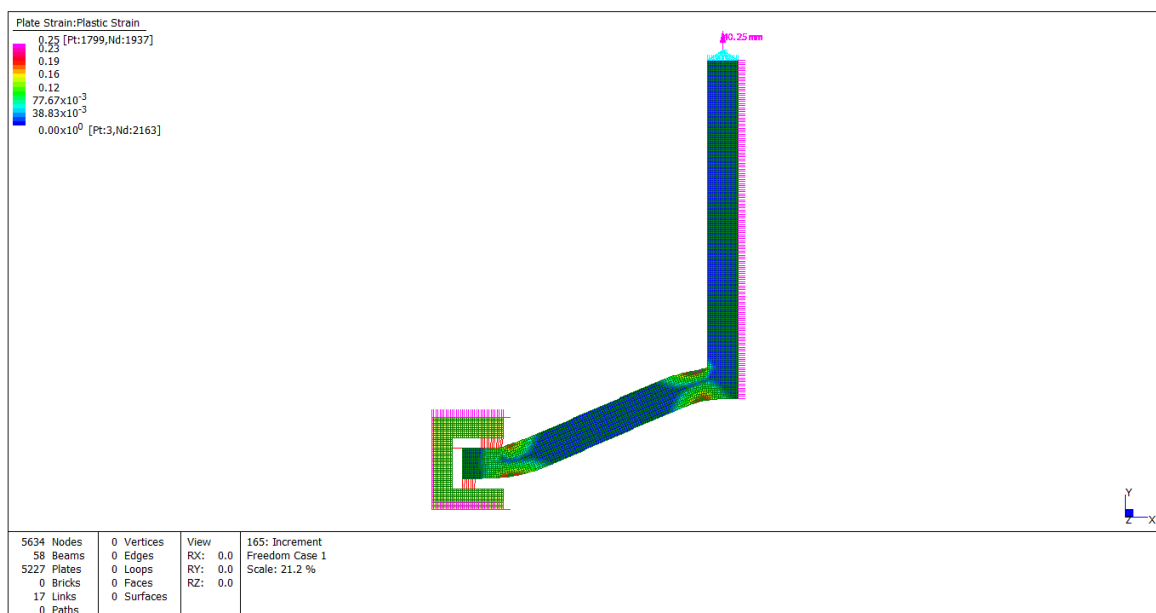


Fig 13. Output of the numerical simulation at ultimate displacement – Plastic strain

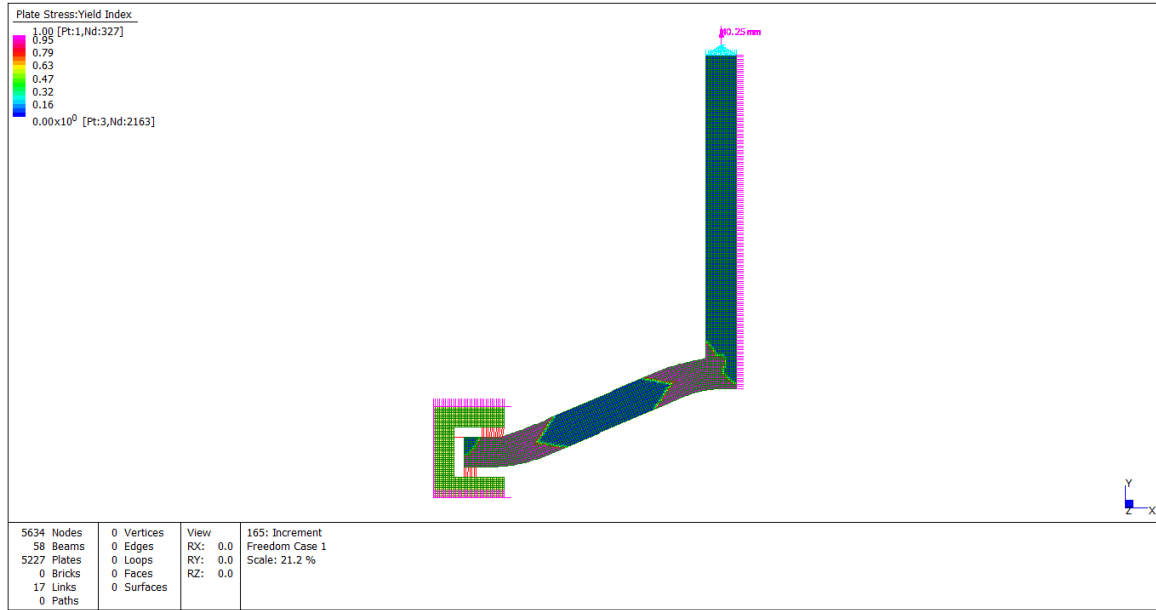


Fig 14. Output of the numerical simulation at ultimate displacement – Yield index

Based on the above numerical results, ductility and behaviour factor can be calculated:

$$\mu = \frac{\delta_u}{\delta_y} = \frac{40.25mm}{0.50mm} = 80.50$$

$$q = \sqrt{2 \times \mu - 1} = \sqrt{2 \times 80.50 - 1} = 12.65$$

The results of the FEA showed that the vertical deformation of the angle at the ultimate limit state, the ductility index and the behaviour factor are approximately in line with the previous analytical estimation.

4.4. Discussion and recommendation on the behaviour factor

The analytical and numerical results of previous sections show that the considered structural glass system, if appropriately design against seismic event, can exhibit significant ductility and it is characterized by large dissipative capacity. It is also observed that remarkable values of behaviour factor can be achieved, even in the case of a simple structural system. More specifically, it is observed that the structural system investigated in this case study exhibits extensive rotation capacity, thus resulting in large dissipated energy, even higher than typical moment resisting frames (for which Eurocode suggest behaviour factor in the range of 4.0-5.5). This is because the typical moment resisting frames referenced by Eurocode consist of highly optimized component (with respect with material/inertia ration, such as I beams/columns), which rotation capacity is smaller than the one computed in this case study for a solid steel plate (See also EN 1998-3 B.5.2 - 2008). From the above considerations, it can be concluded that for structural systems properly design against seismic action, a conservative behaviour factor of 4 could be potentially assumed, which could allow a reduction of the design seismic spectrum by 75%.

The approach presented in this paper could potentially be even more beneficial when it is adopted in complex 3D structures (see for example Fig. 15). In this case, after the plastification of the critical (more stressed) connections and the loss of stiffness due to yielding, additional lateral seismic load migrates to others (underloaded thus stiffer) connections of the structure. The redistribution of the lateral load allows a more efficient structural design, thus a more efficient use of material. This not only allows cost saving, but also relaxes the requirements for the design of the components of the supporting sub-structure.

It is important to mention that the preliminary results provided in this work are mainly intended to evaluate and investigate the potential of such novel energy-based approach for structural glass applications. Detailed studies should be always conducted on a project specific basis. More in general then, future work should focus, on broader parametric studies for general applications of this approach to linear approximate analysis and possible implementation of design recommendations into structural codes and standards. Finally, it is to be recalled that spectrum analysis with behaviour factor, although widely used in structural engineering, are still a very approximate approach for seismic engineering. More sophisticated non-linear analysis should be instead preferred, when possible and appropriate, for a correct structural performance evaluation and safety level assessment.



Fig. 15. Example of 3D complex structural glass system in seismic region

5. Conclusion

This work provides an energy-based approach for efficient design and structural performance evaluation of structural glass systems in seismic regions. Firstly, the analysis methods and the structural performance for structural systems in seismic regions are briefly recalled. Then, the analytical formulation of an energy-based approach for dissipative non-linear structural glass system is described. The analytical equations allow the implementation of the non-linear dissipative behaviour of the structural components into the design of glass systems and to consider appropriate equivalent seismic actions. A simple application is presented by means of some initial analytical and numerical studies. The results show that the combination of appropriate structural design with advanced non-linear analysis allows the achievement of satisfactory performance, even for approximate linear static or dynamic analysis, comparable to the one of other common structural systems in steel or reinforced concrete.

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