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An Analysis of Glass Fracture Statistics

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A numerical method is applied to model the fracture stress and failure location in glass panes subjected to various bending arrangements. The method assumes the weakest-link principle and the existence of surface microcracks. The fracture stress and failure origin are revealed through a search algorithm. The magnitude of strength and the location of fracture are stochastic in nature and can be predicted based on a suitable representation of the surface flaws condition. When the crack size distribution is assumed to be Pareto, the strength distribution is found to be very similar to a Weibull distribution. The stresses in large laterally supported plates which are subjected to uniform pressure are modelled and the distribution of fracture location is determined based on a single population of cracks with a Pareto distributed crack size. Two types of gasket support materials are considered, neoprene and nylon. The softer gasket material produces a greater number of fractures nearer the corners of the plate. A comparison is made with the recorded fracture locations according to various experiments. In addition, a tall vertical panel of laminated glass with a complex geometry and which is subjected to dynamic impact loading is modelled and the distribution of fracture location is determined based on a single population of cracks with a Pareto distributed crack size.

Keywords: Glass, fracture statistics, fracture mechanics, Monte Carlo

1. Introduction

Various models for predicting the fracture stress have been proposed for use on glass (Beason and Morgan 1984, Sedlacek et al. 1999). Some of the models have been implemented in national building codes (DIN 18008-1, ASTM E 1300-04). The failure models proved to have potential for prediction-making within limited domains. However, making accurate predictions of the strength remains a challenge to the general design case of a glass structure with varying boundary conditions and loading types. In fact, large safety factors are implemented in the building codes. Until recently, little attention was paid to the prediction of fracture location. In the following, a method for predicting the failure stress as well as the failure origin of a glass plate subjected to both static and dynamic loading is investigated. The method which assumes the existence of surface microcracks and the governing principle of the weakest-link is applied to different specimen geometries and loading setups. The results are compared with experimental data.

2. Background

The strength of a glass pane can be revealed by subjecting it to bending until it breaks while noting the fracture load (or pressure). The fracture stress at the origin of failure can be calculated assuming that the fracture location is known. The observed fracture stress varies generally within a large range of about 20-200 MPa and is further dependent on a number of factors including the load history, the surface condition (new or weathered or artificially scratched), the size of surface area in tension, the environmental conditions in particular the relative humidity, and the origin of failure, i.e. edge or surface (Mencik 1992).

It has been suggested to use a Weibull distribution for predicting the strength of a structural unit made from annealed float glass (Weibull 1939; prEN 16612:2017). In Eq. (1), the Weibull distribution function for the strength σ is given where k and m denote the scale and shape parameters, respectively.

$$F(\sigma) = 1 - e^{-\left(\frac{\sigma}{k}\right)^m} \tag{1}$$

It has also been suggested to make predictions of the strength based on the Glass Failure Prediction Model (GFPM) (Beason and Morgan 1984). The GFPM was calibrated with experiments in which uniform lateral pressure was applied to full-scale plates with continuous lateral support along all four edges. The American building code ASTM E 1300 implements the GFPM.

The scatter in failure stress magnitude can be explained by assuming that fracture is governed by microscopic surface flaws. Tensile stress is magnified in a localized region near the flaw tip (Griffith 1920). Flaws in glass can cause brittle failure because of the lack in capacity for plastic flow. Surface flaws arise in the production line during manufacture as well as in subsequent handling, transportation, assembly, use, and maintenance. Bulk flaws are disregarded as potential fracture sites in the following.

Variations in the surface condition of glass causes the observed strength to scatter, in general, for some given set of glass specimens, even when identical testing arrangements and specimen geometries are maintained. In fact, experiments have shown that even when the specimens are extracted from the same original standard size plate, the so-called jumbo plate, significant variations in the observed strength remain (Veer et al. 2009; Veer and Rodichev 2011; Vandebroek et al. 2014). Hence, surface flaw characteristics vary significantly not just between plates from different manufacturing batches but also between plates in the same batch.

In this paper, we consider semi-circular cracks that are uniformly distributed over the surface of some glass specimen. The corresponding mode I stress intensity factor (SIF) is determined using the following equation with *a* referring to the crack depth (Irwin 1957; Newman and Raju 1981)

$$K_I = 1.14\frac{2}{\pi}\sqrt{\pi a}\sigma_n \tag{2}$$

In Eq. (2), σ_n is the tensile stress normal to the crack plane. The mode I fracture criterion is

$$K_I \le K_{Ic} \tag{3}$$

and it is assumed that the fracture toughness K_{Ic} equals to 0.75 MPa m^{½} (Mencik 1992). It is assumed that the individual cracks do not interact with each other. As a mixed mode criterion we take

$$\sqrt[4]{K_I{}^4 + 6(K_I{}^2 + K_{II}{}^2) + K_{II}{}^4} \le K_{Ic} \tag{4}$$

which is based on the maximum non-coplanar energy release rate (Hellen and Blackburn 1975), see also Thiemeier et al. (1991). In Eq. (4), K_{II} can be approximated as (Thiemeier et al. 1991)

$$K_{II} = 1.14\frac{4}{\pi}\frac{1}{2-\nu}\sqrt{\pi a}\tau \tag{5}$$

with ν referring to Poisson's ratio and τ the shear stress in the crack plane.

According to experimens with Hertzian indentation fracture in glass, flaw size can be closely fitted by a Pareto distribution (Poloniecki and Wilshaw 1971; Tandon et al. 2013). The Pareto distribution is (Forbes et al. 2010)

$$F(a) = 1 - \left(\frac{a_0}{a}\right)^c \tag{6}$$

where the scale and shape parameters are a_0 and c, respectively. It has been demonstrated that the Weibull distribution function can be derived from the WLP while assuming that the surface flaws condition is represented by a single population of cracks with a crack depth that is Pareto distributed (Jayatilaka and Trustrum 1977). It is then supposed that the stress state is uniform tensile and that the crack planes are oriented normal to the uniaxial stress. Let $f(a)$ denote the probability density function of the crack depth. Then the probability of failure at stress σ for a single crack is

$$F(\sigma) = \int_{a_c}^{\infty} f(a)da \tag{7}$$

where a_c is the critical crack depth that prompts failure for a crack subjected to tensile stress perpendicular to the crack plane. The critical crack depth is obtained through combination of Eqs. (2) and (3)

$$a_c = \frac{K_{Ic}{}^2}{Y^2\pi\sigma^2} \tag{8}$$

where for the sake of convenience, the geometry factor Y has been substituted for. The geometry factor is in this case given by

$$Y = 1.14\frac{2}{\pi} \tag{9}$$

Supposing that crack depth is Pareto distributed, we derive from Eqs. (6) and (7) while substituting for Eq. (8) that

$$F(\sigma) = \left(\frac{Y\sqrt{\pi a_0}\sigma}{K_{Ic}}\right)^{2c} \tag{10}$$

For N cracks, the probability of failure, P_f, is given by the following equation, supposing the WLP

$$P_f = 1 - \left(1 - F(\sigma)\right)^N \tag{11}$$

When N is large, Eq. (11) can be approximated by the following equation which can be shown by performing a Taylor series expansion

$$P_f = 1 - \exp\left(-NF(\sigma)\right) \tag{12}$$

so that for large N, we have approximately

$$P_f = 1 - \exp\left(-\left(\frac{Y\sqrt{\pi a_0}N^{\frac{1}{2c}}}{K_{Ic}}\sigma\right)^{2c}\right) \tag{13}$$

Eq. (13) can be simplified to Eq. (1), i.e. the Weibull distribution function, with the scale parameter

$$k = \frac{K_{Ic}}{Y\sqrt{\pi a_0}N^{\frac{1}{2c}}} \tag{14}$$

and the shape parameter

$$m = 2c \tag{15}$$

Hence, it is possible to calculate the distribution of macroscopic strength of a stressed solid by starting from an analysis of the microscopic defects and applying the WLP. Others who have considered this include e.g. Matthews et al. (1976) and Batdorf and Heinisch (1978). However, the mathematics soon become intractable as various assumptions are made for the stress state, fracture criterion, crack size distribution, flaw density, crack plane orientation, and the existence of multiple flaw populations.

Stress corrosion causes subcritical crack growth when the glass is stressed in tension in an ambient atmosphere which relates, in particular, to the relative humidity being greater than zero (Charles 1958a, 1958b). However, subcritical crack growth is only observed when the mode I SIF exceeds a threshold limit value estimated at about 0.25 MPa $m^{1/2}$ (Wiederhorn and Bolz 1970). In this paper the effect of stress corrosion is neglected.

3. Numerical method

Yankelevsky (2014) proposed a numerical solution method for calculating the strength distribution of a brittle solid that starts from an analysis of the microscopic defects. The weakest-link principle was applied in Monte Carlo simulations with Griffith flaws to model the fracture stress and fracture location of square glass plates subjected to bending. In Yankelevsky (2014), the plates were laterally supported along two opposite edges and subjected to a line-load at midspan. A Monte Carlo simulation was carried out for a large sample of 5000 virtual specimens. The method offers a tractable way to calculate the distribution of strength and fracture location for arbitrary stress states, fracture criteria, crack plane orientations, and crack size distributions, while allowing for the implementation of multiple flaw populations. The standard size so-called jumbo plate which measures 3.21x6.00 m^2 is taken as a starting point. The surface area is divided into unit cells and cracks are distributed over the cells according to a uniform distribution. It is supposed that the orientation of the crack plane is uniformly distributed. In this study, the total number of cracks on the jumbo plate is fixed and depends on the flaw density. It is assumed for the model that the flaw density is 2 cm^{-2} (Wereszczak et al. 2014). The sampled crack size is based on some statistical distribution, e.g. the Pareto distribution. The random flaws are resampled each time a new jumbo plate is modelled. In summary, the stochastics of the flaws comprise the location, the crack plane orientation, and the size. A specimen is extracted from the jumbo plate and analyzed. The analysis depends on a comparison of the cracks with the time-dependent stress field using fracture mechanics. However, the stress distribution over time only needs to be calculated once for any given specimen type and bending arrangement. It is the distributed set of cracks that is resampled in each new simulation of the glass fracture. Fracture is prompted when the SIF envelope for the first time intersects with the fracture toughness. When this happens, the fracture stress and location can be determined based on the first unit cell that contains a failing crack.

In a recent paper (Kinsella and Persson 2018), this type of numerical method was applied to model the fracture stress and failure location of small glass plates subjected to double ring bending. The results allowed for making comparisons between different fracture criteria. Furthermore, a dual population concept of flaws was fitted to model the fracture stress in an empirical data set, the purpose of which was to model the apparent bimodality in the fracture stress distribution (Simiu et al. 1984). Glass fracture data tends to exhibit bimodalities (Veer et al. 2009).

This kind of numerical method was also used by Pathirana et al. (2017) who implemented a dual population concept in Monte Carlo simulations of Griffith flaws for the determination of the strength distribution in panels subjected to point contact actions.

4. Application to laterally supported plates subjected to uniform pressure

In this paper, the results from new simulations are presented that were carried out using the numerical method described in Sec. 3. The results pertain to laterally supported plates subjected to uniform pressure. As a background, the following is noted. Bending tests that record the fracture location in new full-scale plates which are laterally supported along all four edges and subjected to uniform pressure have previously been carried out by Johar (1981, 1982), Kanabolo and Norville (1985), and Calderone (1999). In Johar's and Kanabolo and Norville's experiments, the glass plates were supported between (approximately) 6 mm wide neoprene gaskets. In Calderone's experiment, 20 mm thick nylon gaskets were used. The plate nominal thickness was 6 mm in all experiments whereas the average thickness was 5.8 mm. Tab. 1 lists the sample sizes as well as the relative frequency of surface failures to edge failures. In Tab. 1, only those failures which were unambiguously identified as originating from either the surface or the edge were included in the statistics. In other words, when there was recorded multiple potential fracture origins which included a mixture of surface and edge sites, these were not counted and included in the Tab. 1 statistics. This was done for the sake of consistency because it is generally believed that the edge condition and hence the edge strength differs from the surface condition. Fig. 1 shows the recorded fracture locations and depicts the various plate dimensions that were used in the experiments.

Two square plates measuring 1200x1200 mm^2 and with a thickness of 5.8 mm were modelled using the FEM with ABAQUS/CAE (2013). The plates were laterally supported along all four edges between continuous 6 mm wide gaskets which were 6 mm in thickness. In one case the gasket material was neoprene (Shore A55) and in the other case it was nylon. The neoprene was modelled as an incompressible Neo-Hookean hyperelastic material with shear modulus G=1 MPa (Gent 2012). The nylon was modelled as an isotropic linear elastic material with Young's modulus E=3 GPa and Poisson's ratio ν=0.34. The gaskets were rigidly supported on the side opposite to the contact surface with the glass. A friction coefficient of 0.19 was adopted for the contact between gasket and glass. The glass material was assumed to have a Young's modulus E=72 GPa and a Poisson's ratio ν=0.23. Solid-shell elements were used for the glass part while employing a quadrilateral mesh generator. Hybrid elements were used for the hyperelastic material parts. In the case of the neoprene material, an adaptive meshing technique was employed for the gasket parts to improve the convergence. For symmetry reasons only one quarter of the plate was modelled. The plate was subjected to uniform lateral pressure. Fig. 2 shows the deformed state of the plate as seen from one corner when the gasket material was neoprene. Figs. 3 and 4 show the maximum in-plane principal stresses on the "tension" and "compression" sides of the plate, respectively, for both plates at a pressure magnitude of 40 kPa. The maximum tensile stress at this pressure was 97 MPa (nylon) and 164 MPa (neoprene), respectively, on the "tension" side, and 165 MPa (nylon) and 48 MPa (neoprene), respectively, on the "compression" side. The "tension" side refers to the side of the plate that is in tension at the centre point. The results show that with the softer gasket material, the tensile stresses concentrated nearer towards the edges of the plate. In fact, on the "tension" side, the maximum tensile stress was also significantly greater in this case. However, with the harder gasket material, it was observed that on the "compression" side, there is a very high build-up of tensile stress near the edges.

Table 1: Sample size and relative frequency of surface to edge failure in experiments with laterally supported plates subjected to uniform pressure. Some data points were excluded in the case of multiple potential fracture locations which contained a mixture of surface and edge failure sites.

Reference	Total no. of failures	No. of surface fail's	Rel. freq. of surf. fail's
Johar (1981)	78	54	0.69
Johar (1982)	106	71	0.67
Kanabolo and Norville (1985)	206	152	0.74
Calderone (1999)	195	152	0.78

The strength and fracture locations were simulated using the numerical method that was described in Sec. 3. It was assumed that the surface condition is characterized by a single population of semi-circular cracks with a Pareto distributed crack depth. The Pareto scale and shape parameter values were chosen as a_0=4 µm and c=3.0, respectively, cf. Eq. (6). The cracks were uniformly distributed over the surface area and the unit cell size was 5x5 mm^2. The crack density was 2 cm^{-2}. The motivation behind the choice of Pareto distribution parameter values comes from assuming a Weibull distribution for the strength with the parameter values k=74 MPa and m=6. Eq. (14) and (15) then give (approximately) the said Pareto parameter values with N=5655. In fact, this Weibull distribution gives a characteristic value of the bending strength $\sigma_{b,ch}$=45 MPa defined as the 5% fractile, cf. Sedlacek et al. (1999). According to Haldimann (2006), this Weibull distribution represents the breakage stress of new glass plates in R400 double ring bending tests at a stress rate of 2 MPa s^{-1} the tests of which were conducted as a basis for the DIN 1249-10:1990. With an assumed flaw density of 2 cm^{-2} the number N=5655 is obtained because the stressed area within the loading ring is 0.2827 m^2.

Figs. 5 and 6 show the simulated fracture locations based on a series of 5000 simulations each for the two types of gaskets, i.e. neoprene and nylon. In Fig. 5, the fracture criterion that was used assumes that the crack planes are oriented normal to the maximum principal stress, whereas in Fig. 6, the mixed mode fracture criterion, Eq. (4), was used. In this case, it was assumed that the crack plane angles were uniformly distributed in $[0,\pi)$.

Fig. 7 depicts the distribution in fracture stress for both types of gasket materials while assuming a mode I criterion with the crack planes oriented perpendicular to the maximum principal tensile stress. A two-parameter Weibull distribution was fitted to the data samples and is also shown in the diagrams. It can be noted that the mean fracture stress is slightly lower with the mixed mode fracture criterion.

5. Application to tall panels subjected to impact load

The dynamic impact load case is often relevant when performing a strength design of a glass structure. With an accurate description of the stress distribution in the impacted pane, it is possible to predict the likely fracture location. However, it is not necessarily the case that the failure location coincides with the maximum principal tensile stress (Natividad et al. 2016). By implementing the numerical method described in Sec. 3 it is possible to model the distribution of fracture location. The European standard EN-12600 details a method for testing glass to classify it in terms of impact strength.

The distribution in fracture location was studied for a tall vertical panel subjected to an impact load. The panel consists of a laminated unit with two glass plies. The panel measures approximately 1x4 m^2 in surface area and each ply has a thickness of 10 mm. The full transient FE simulation of the panel and impactor were based on a previous model which is described in Fröling et al. (2014). The panel was supported on two sides (top and bottom edges) and it had a 6x6 array of ventilation holes near the bottom edge, cf. Fig. 8a for an illustration. The impactor consists of a weight encased in two tyres, the weight of the impactor being 50 kg according to standard (EN-12600). The tyre was swung into the panel in a pendulum motion thus generating a soft impact with a long pulse time. The glass and PVB interlayer parts were modelled by means of a hexahedral solid-shell element. The rubber supports were modelled using a solid element. The glass, interlayer and supports were modelled as linear elastic materials and the material parameters which were adopted from Persson and Doepker (2009) and prEN 16612:2017 are shown in Tab. 2. The initial velocity of the impactor was 2.97 m s^{-1} which corresponds to a fall height of 0.450 m. The centrical impact occurred at a height of 1.2 m.

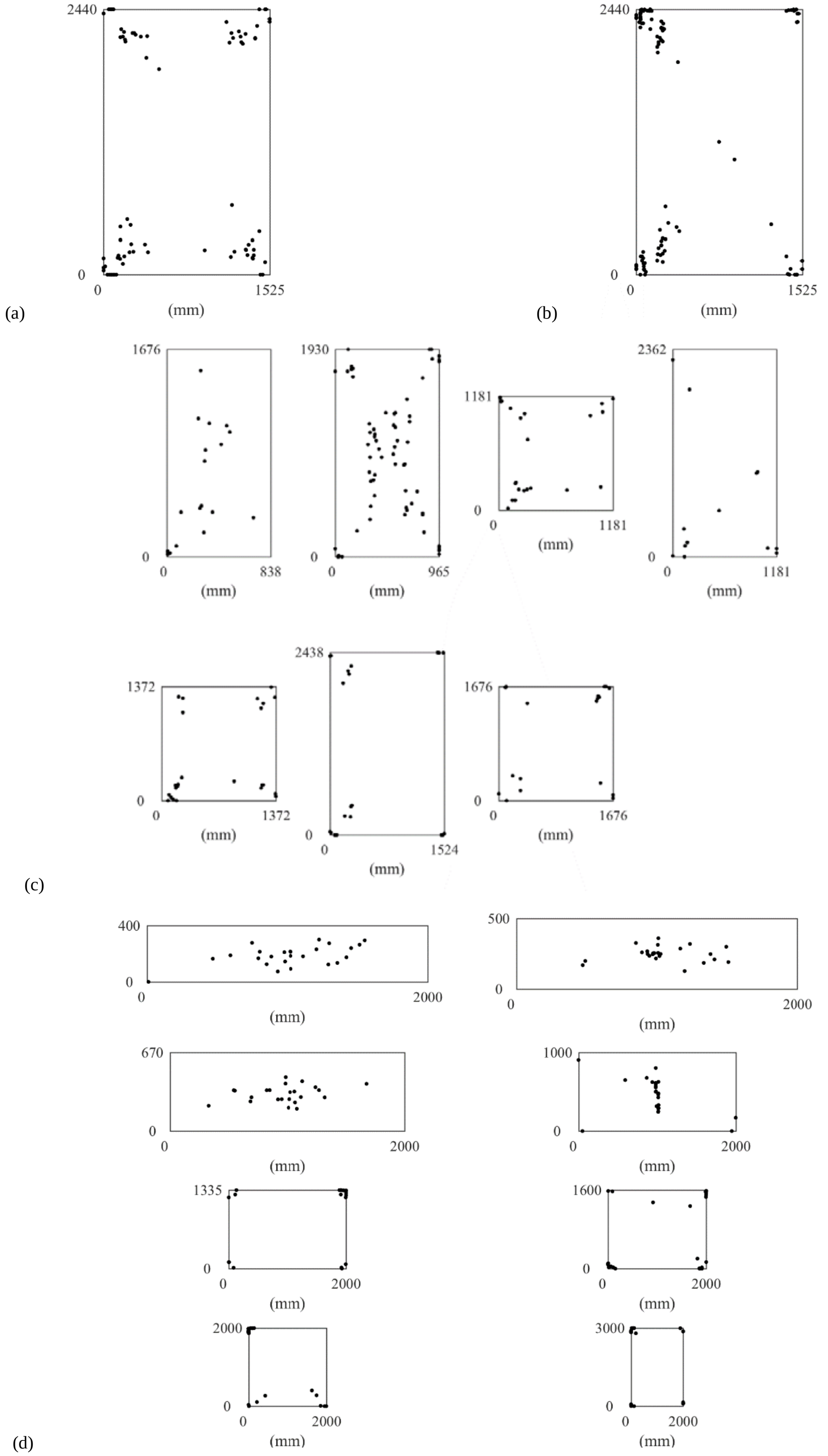


Fig. 1 Fracture origins according to four experiments (a) Johar (1981), (b) Johar (1982), (c) Kanabolo and Norville (1985), and (d) Calderone (1999).

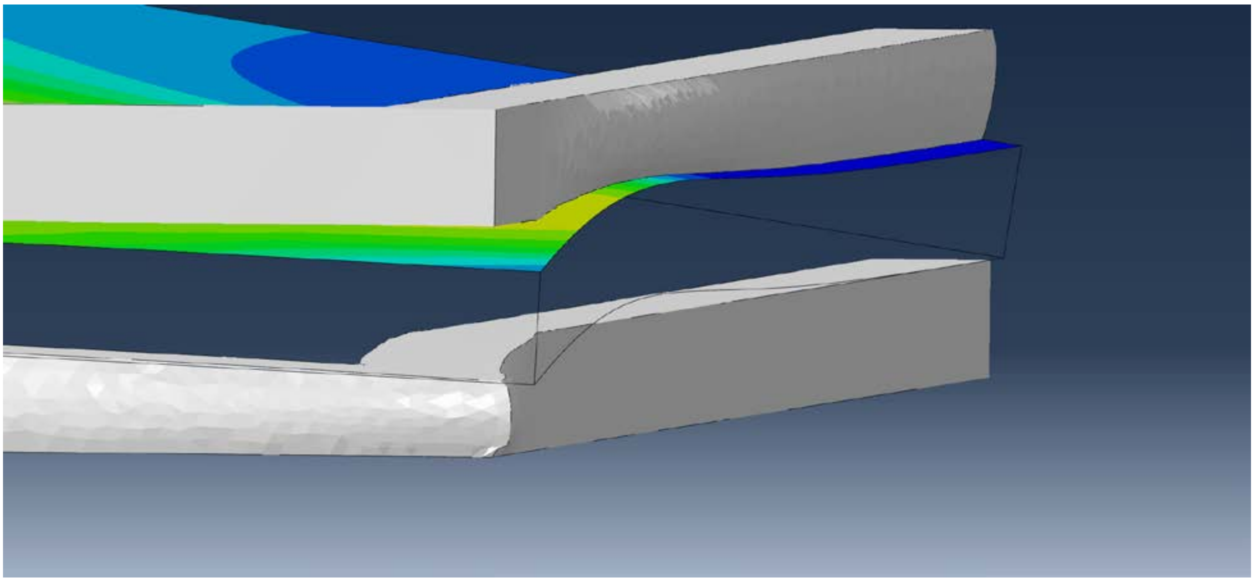


Fig. 2 Deformed state of a plate which is supported laterally between neoprene gaskets and subjected to uniform pressure. As seen from one corner. For symmetry reasons only one quarter of the plate is visible.

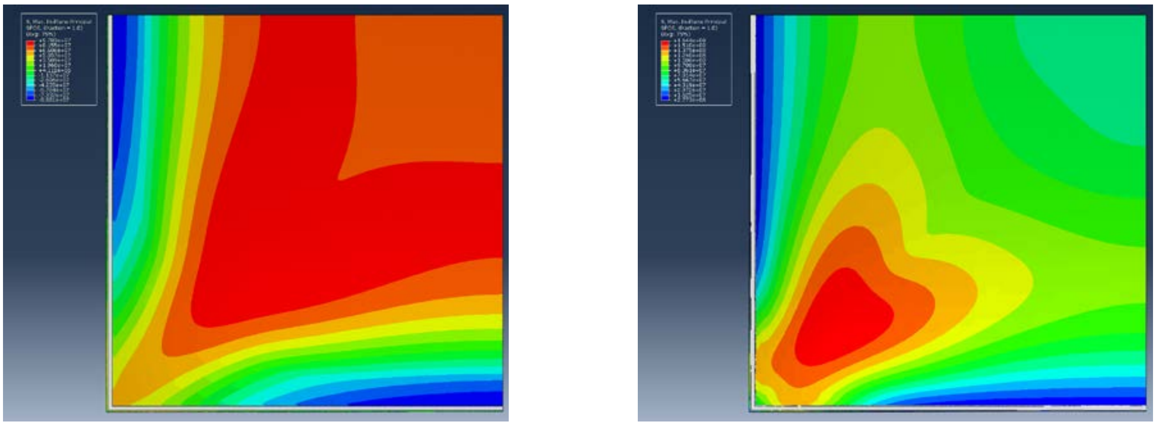


Fig. 3 Stress contours (maximum in-plane principal) on the “tension” side of the plate with (left) nylon gaskets and with (right) neoprene gaskets at the lateral pressure magnitude 40 kPa.

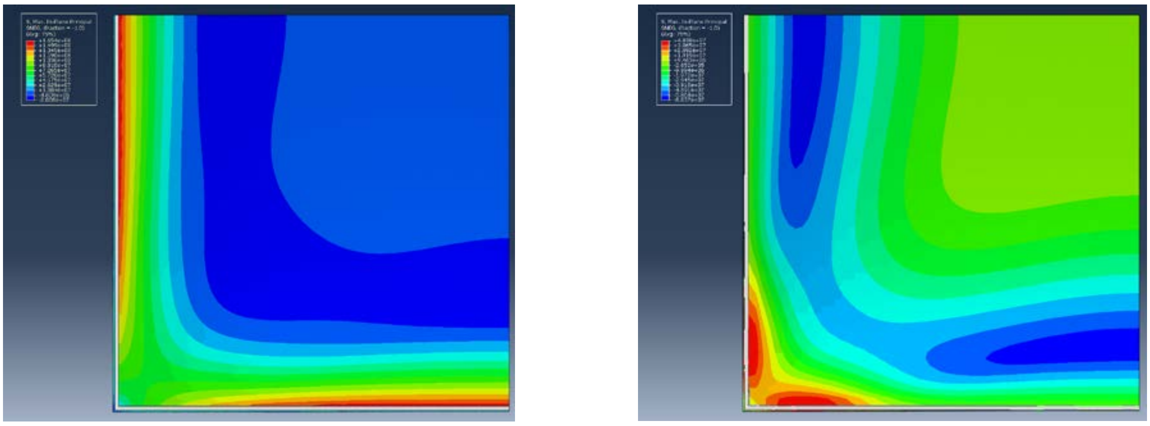


Fig. 4 Stress contours (maximum in-plane principal) on the “compression” side of the plate with (left) nylon gaskets and with (right) neoprene gaskets at the lateral pressure magnitude 40 kPa.

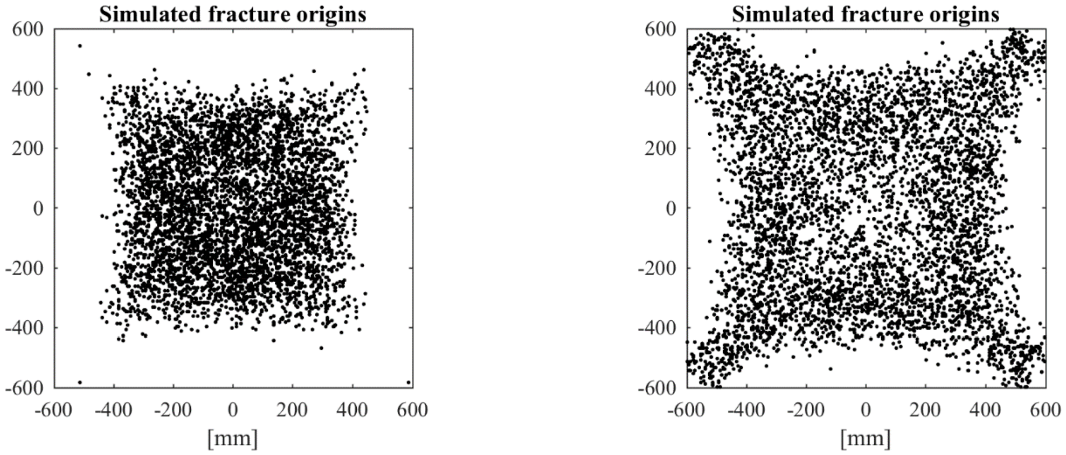


Fig. 5 Simulated fracture locations in the case of (left) nylon gaskets and (right) neoprene gaskets with a pure mode I fracture criterion assuming all crack planes to be oriented perpendicular to the max. princ. stress.

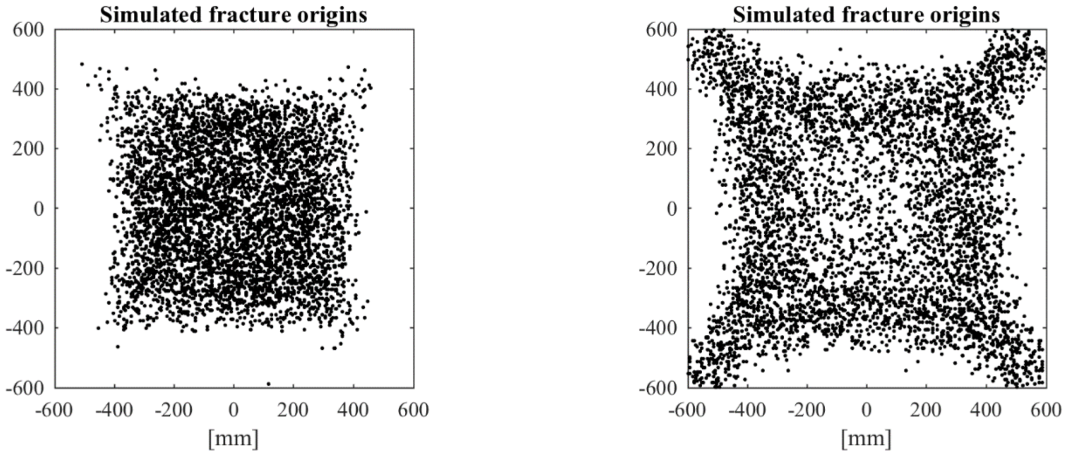


Fig. 6 Simulated fracture locations in the case of (left) nylon gaskets and (right) neoprene gaskets with a mixed mode fracture criterion.

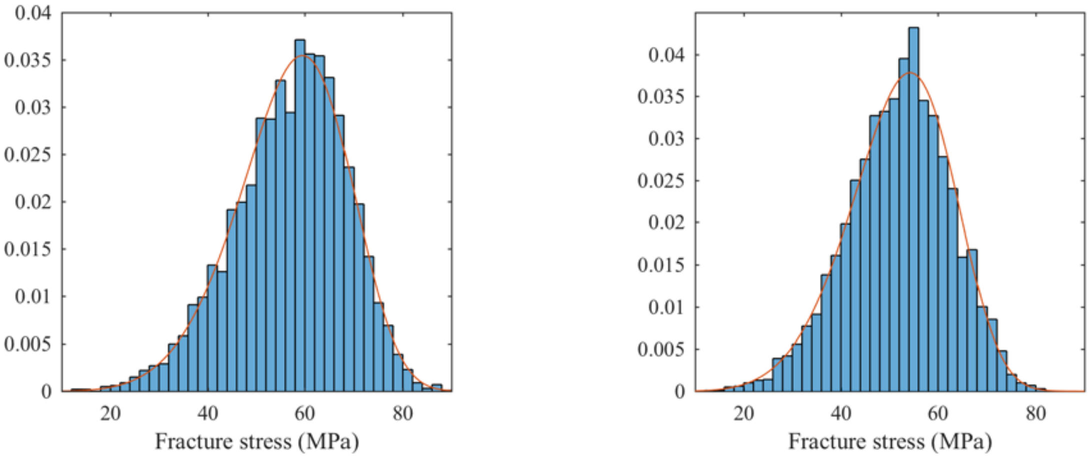


Fig. 7 Simulated fracture stress in the case of (left) nylon gaskets and (right) neoprene gaskets with the MPTS fracture criterion, i.e. assuming all crack planes to be oriented perpendicular to the max. princ. stress. Solid line corresponds to a fitted Weibull distribution. The histograms are normalized to reflect a probability density.

Fig. 8b shows the maximum in-plane principal stress contours at time equal to 15 ms when the maximum stress was about 45 MPa. Fig. 8c shows the stress contours at time equal to 30 ms when the maximum stress had reached about 82 MPa. The greatest stress (82 MPa) was located near the top row of ventilation holes. Fig. 9a shows the distribution of fracture location using a mode I fracture criterion without consideration of crack plane orientation, i.e. assuming that all crack planes are oriented normal to the maximum principal tensile stress. It was assumed that the surface condition is represented by a single population of cracks with a Pareto distributed depth with parameter values a_0=4 µm and c=3.0, and that the crack density is 2 cm^{-2}. Fig. 9a depicts in total 989 fractures which occurred during the simulation of 5000 virtual panel impacts. About 40% of the failures in total occurred near one of the ventilation holes. The area near a ventilation hole was in this case defined by a bounding box around the whole 6x6 array. Fig. 9b shows the resulting strength distribution which is not necessarily in agreement with a Weibull distribution.

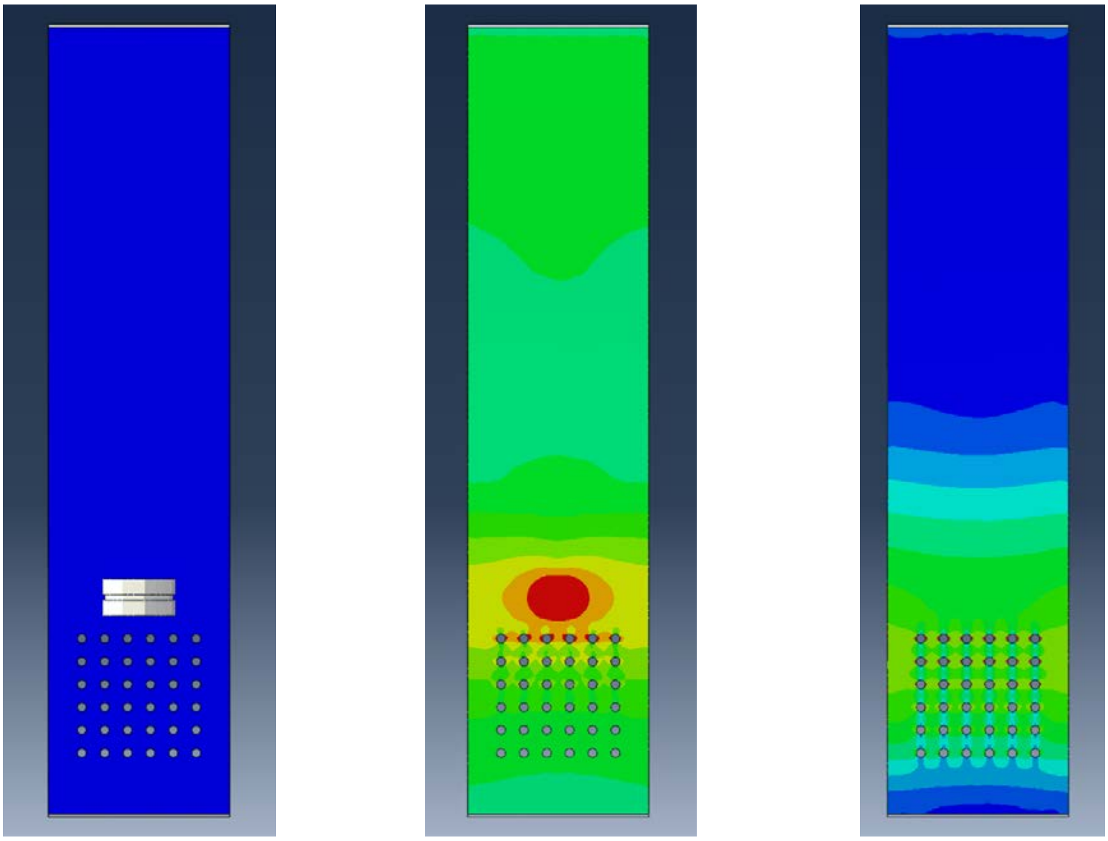
Table 2: Material parameters

Material	E (MPa)	ν	ρ (kg m^{-3})
Glass	70000	0.2	2500
PVB interlayer	180	0.49	1250
Rubber support	15	0.44	1250
Impactor	2	0.3	900

6. Discussion

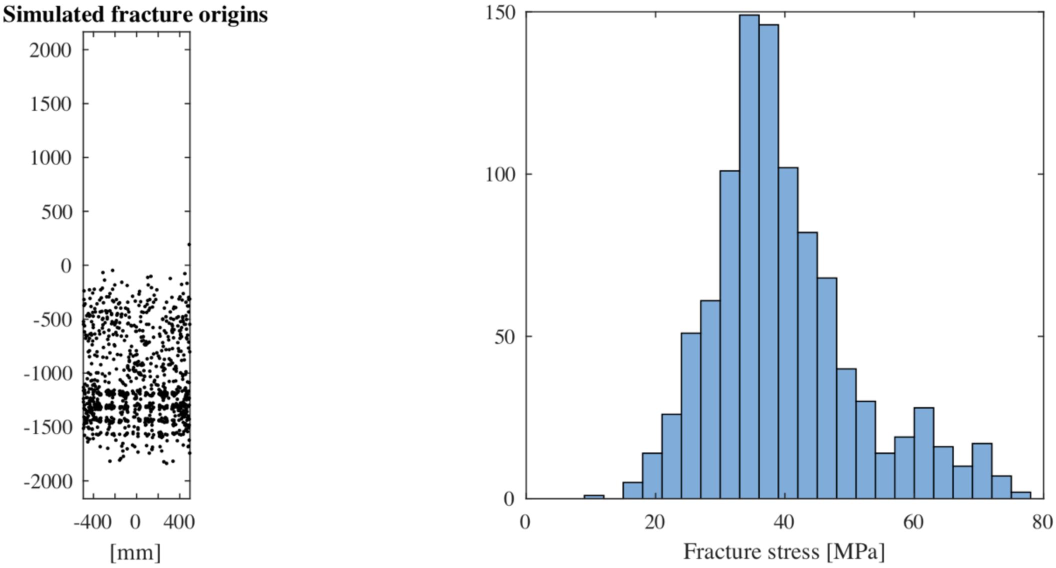
In theory, brittle fracture in glass is promoted by the existence of a large set of surface microcracks with a location and size distribution that can be described using some random variable. Because of the limited capacity for plasticity in glass, the failure mode is governed by the WLP, i.e. the first fracturing flaw prompts global breakage. A failure prediction model that is consistent with theory must therefore take into account the existence of surface microcracks including the stochastics of these, and the WLP. The Weibull model adopts the WLP and can, in theory, be associated with a single population of surface cracks having a Pareto distributed crack size. The Weibull model is preferred in major standards including the European draft prEN 16612:2017. However, the Weibull models that are fitted to empirical data are so different in scale and shape that is hard to predict the strength in general while adopting this type of distribution. A similar limitation appears to apply to the GFPM of which it has been said that it "is best suited to representing glass strength for specific test conditions." (Reid 2007) As a matter of fact, it is not just the fracture stress magnitude that scatters, the failure location is also variable. It has been shown that the fracture origin rarely occurs at the point of MPTS in laterally supported plates subjected to uniform out-of-plane loading (Natividad et al. 2016).

The method which was investigated in this paper offers a promising alternative to the ordinary Weibull model for use in failure prediction of structural glass units. Firstly, the method is based on the physics of brittle fracture. A representation of the surface condition is implemented and fracture mechanics are combined with the WLP to reveal the breakage stress and location. By assuming that the surface condition is represented by a single population of cracks with a Pareto size distribution, it is possible to obtain a Weibull distribution for the strength. The new model differs from the Weibull model in that a greater freedom is afforded towards the representation of the surface condition in glass. Now, the available data on the surface condition is scarce. As current techniques are improved, and new methods are developed to probe the surface condition, more reliable data can be supplied as input to this kind of failure model. It is moreover possible to evaluate failure based on different fracture criteria including mixed mode criteria in a way that would be more tractable than while using the ordinary Weibull distribution. The mathematics soon become intractable when evaluating the analytical expressions necessary to implement different fracture criteria, cf. e.g. Batdorf and Heinisch (1978). With the new method, it is possible to control the crack plane orientations in a way that would not be feasible using the ordinary Weibull distribution. If one for instance assumes that the crack planes lie in some particular direction on certain parts of the surface due to, for example, mechanical abrasion, then it would be quite possible to implement this in the new model through a suitable setup of the surface condition. It is also possible to implement multiple flaw populations. The new method offers the possibility to predict the fracture location which can be useful in certain situations. For example, glass structures with more complicated geometry containing corners and holes, and glass structures subjected to more advanced loading situations such as uneven static loading and dynamic loading.



(a) (b) (c)

Fig. 8 (a) Tall panel and soft impactor. (b) Stress contours (max. in-plane princ.) when the max stress had reached 45 MPa at time 15 ms. (c) Stress contours when the max stress had reached 82 MPa at time 30 ms. NB, maximum stress in (c) is near the edges of the top row of ventilation holes. Red colour corresponds to tensile stress.



(a) (b)

Fig. 9 (a) Simulated fracture origins. (b) Strength distribution.

In the present study, a method was applied to model the strength and fracture location of laterally supported plates subjected to uniform pressure. The comparison of the empirical data appears to indicate that a significant portion of failures in tests of large plates occur near or on the edges. This might indicate that failure is sensitive to shear stress. According to one study (Reid 2007), a series of 59 small specimens of annealed glass plates generated unexpected results when tested in a double ring bending device. The proportion of failures outside the loading ring was much greater than expected based on Weibull statistics which do not factor shear stress into the failure criterion. There is no shear stress in the loading ring area because the stress state is equibiaxial. In the case of a large laterally supported plate subjected to uniform pressure, there emerges shear stress near the corners of the plate. In this paper, such large plates were investigated while using both a mode I fracture criterion and a shear sensitive mixed mode criterion. However, from the simulation results, it is hard to see a significant impact on the fracture location due to the presence of shear stress near the corners of the plate. Nevertheless, the fracture origin was increasingly located nearer the corners when the support gaskets were made from a softer material, i.e. neoprene. As the Figs. 3 and 4 show, the tensile stress on the “compression” side of the plates are significant, especially in the case with the nylon gaskets. At the applied pressure 40 kPa, the maximum tensile stress on the “compression” side of the nylon supported plate was in fact on par with the maximum tensile stress on the “tension” side of the neoprene supported plate. This implies that a thorough analysis of the failure of laterally supported plates subjected to uniform pressure should consider both faces of the plate. This was not done in the present study but could be conducted in a future investigation. However, according to one study on large plates subjected to uniform loading (Calderone 1999), there were only two fractures occurring from the “compression” side of 195 specimens tested in total corresponding to a relative frequency of about 1%. In that study, nylon gaskets were used and the glass was fixed firmly between the nylon supports. This indicates that failures from the “compression” side are unlikely in practical situations. However, further investigation is required in order to verify this. More important perhaps, is the fact that a significant proportion of failures occur from the edges according to experimental data, cf. Tab. 1. In the modelling that was done in connection with this paper, only the surface condition in glass was considered. The edge condition was not represented separately. This is an important issue, however, that might be considered in future research work.

The case with the vertical panel impacted by a soft body illustrates how the new method can be applied to model specimens with a more complex geometry subjected to dynamic loading. This loading leads to a time-dependent stress distribution that initially affects a relatively large portion of the glass surface to moderate tensile stress and subsequently a much smaller portion is affected, in particular at the ventilation holes, to higher tensile stress. Even if the strength distribution is known a priori, i.e. a Weibull distribution, the question remains as to how the fracture location is distributed. The simulations which were carried out show that ultimately about 40% of the failures occurred near the holes. However, the edge condition in glass is very relevant in this case and should perhaps be represented differently than the surface condition. Further research needs to be conducted in order to properly model this load case while taking the edge condition into consideration. In the simulation of the panel, stress corrosion was not considered. However, in this particular case, the dynamic impact load produces a very high stress rate. In fact, the overall maximum tensile stress was reached within about 30 ms which corresponds to an average stress rate of approximately 2700 MPa s^{-1}. Presumably, any effects of static fatigue would be limited because there would be very little time for stress corrosion to take place. It is therefore believed that stress corrosion in this case would have only a negligible effect on the results. Interestingly, Haldimann (2006) carried out experiments on glass plates which were loaded at both low and very high stress rates (0.2 MPa s^{-1} and 21 MPa s^{-1}, respectively) and compared the results. His findings seemed to indicate that the behaviour of a specimen subjected to a stress rate of as much as 21 MPa s^{-1} in ambient conditions nearly approaches that of a specimen in inert conditions.

7. Conclusions

The distribution of fracture stress and failure location in glass can be modelled using a numerical method that is based on well-established concepts including the WLP and the existence of surface microcracks. The method is applied to model the strength and fracture origin in large laterally supported plates subjected to uniform pressure and in a tall panel with a complex geometry that is subjected to impact loading. By assuming that the surface condition is represented by a single population of cracks with a Pareto distributed crack size it is possible to obtain a strength distribution that is similar to a Weibull distribution. As current methods are refined and new techniques are developed to probe the surface condition of glass, this new numerical tool has potential for greater versatility in modelling glass fracture statistics since it allows for various surface flaws conditions and fracture criterions to be used.

References

ABAQUS/CAE: Version 6.13. Dassault Systèmes, (2013)
ASTM E 1300-04: Standard Practice for Determining Load Resistance of Glass in Buildings. (2004)
Batdorf, S.B., Heinisch, H.L.: Weakest Link Theory Reformulated for Arbitrary Fracture Criterion. J Am Ceram Soc **61**(7-8), 355-358 (1978)
Beason, W.L., Morgan, J.R.: Glass Failure Prediction Model. J Struct Eng **110**, 197-212 (1984)
Calderone, I.J.: The equivalent wind loading for window glass design. Monash University (1999)
Charles, R.J.: Static Fatigue of Glass I. J Appl Phys **29**, 1549-1553 (1958a)
Charles, R.J.: Static Fatigue of Glass II. J Appl Phys **29**, 1554-1560 (1958b)
DIN 18008-1:2010: Glas im Bauwesen - Bemessungs- und Konstruktionsregeln - Teil 1: Begriffe und allgemeine Grundlagen. (2010)
DIN 1249-10:1990: Flachglas im Bauwesen - Teil 10: Chemische und physikalische Eigenschaften. (1990)
EN 12600: Glass in building - Pendulum test - Impact test method and classification for flat glass. CEN (2003)
Forbes, C., Evans, M., Hastings, N., Peacock, B.: Statistical distributions. John Wiley and sons, Inc., (2010)
Fröling, M., Persson, K., Austrell, P.E.: A reduced model for the design of glass structures subjected to impact loads. Engineering Structures **80**, 53-60 (2014)
Gent, A.: Engineering with Rubber. Hanser, (2012)
Griffith, A.A.: The Phenomena of Rupture and Flow in Solids. Phil. Trans. R. Soc. **A221**, 163-163 (1920)

Haldimann, M.: Fracture strength of structural glass elements -- analytical and numerical modelling, testing and design. Ecole Polytechnique Fédérale de Lausanne EPFL (2006)

Hellen, T.K., Blackburn, W.S.: The Calculation of Stress Intensity Factors for Combined Tensile and Shear Loading. Int J Fract **11**, 605-605 (1975)

Irwin, G.R.: Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate. J. Appl. Mech. **24**, 361-361 (1957)

Jayatilaka, A.D., Trustrum, K.: Statistical approach to brittle fracture. J Mater Sci **12**, 1426-1430 (1977)

Johar, S.: Dynamic fatigue of flat glass - Phase II. In. ORF, (1981)

Johar, S.: Dynamic fatigue of flat glass - Phase III. In. ORF, (1982)

Kanabolo, D.C., Norville, H.S.: The strength of new window glass using surface characteristics. Texas Tech University (1985)

Kinsella, D.T., Persson, K.: A Numerical Method for Analysis of Fracture Statistics of Glass and Simulations of a Double Ring Bending Test. Glass Struct & Eng (2018). doi:10.1007/s40940-018-0063-z

Matthews, J.R., McClintock, F.A., Shack, W.J.: Statistical determination of surface flaw density in brittle materials. J. Am. Ceram. Soc. **59**, 304-308 (1976)

Mencik, J.: Strength and Fracture of Glass and Ceramics, vol. 12. Elsevier, (1992)

Natividad, K., Morse, S.M., Norville, H.S.: Fracture Origins and Maximum Principal Stresses in Rectangular Glass Lites. Journal of Architectural Engineering **22**(2), 04015014 (2016). doi:10.1061/(ASCE)AE.1943-5568.0000197

Newman, J.C., Raju, I.S.: An empirical stress intensity factor equation for the surface crack. Eng. Fract. Mech. **15**, 185-192 (1981)

Pathirana M., Lam N., Perera S., Zhang L., Ruan D., Gad E.: Risks of failure of annealed glass panels subject to point contact actions. Int J Solids Struct **129**, 177–194 (2017)

Persson, K., Doepker, B.: Glass panes subjected to dynamic impact loads. Proc. of the XXIV A.T.I.V. Conference, Parma, 9-10 July (2009)

Poloniecki, J.D., Wilshaw, T.R.: Determination of Surface Crack Size Densities in Glass. Nature Physical Science **229**, 226-227 (1971)

prEN 16612:2017: Glass in building - Determination of the lateral load resistance of glass panes by calculation. CEN, (2017)

Reid, S.G.: Effects of spatial variability of glass strength in ring-on-ring tests. Civ Eng Environ Syst **24**, 139-148 (2007)

Sedlacek, G., Blank, K., Laufs, W., Güsgen, J.: Glas im Konstruktiven Ingenieurbau. Ernst & Sohn, (1999)

Simiu, E., Reed, D.A., Yancey, C.W.C., Martin, J.W., Hendrickson, E.M., Gonzalez, A.C., Koike, M., Lechner, J.A., Batts, M.E.: Ring-on-ring tests and load capacity of cladding glass. In. (1984)

Tandon, R., Paliwal, B., Gibson, C.: Practical aspects of using Hertzian ring crack initiation to measure surface flaw densities in glasses: influence of humidity, friction and searched areas. Philos Mag **93**, 2847-2863 (2013)

Thiemeier, T., Brückner-Foit, A., Kölker, H.: Influence of the fracture criterion on the failure prediction of ceramics loaded in biaxial flexure. J. Am. Ceram. Soc. **74**(1), 48-52 (1991)

Vandebroek, M., Louter, C., Caspeele, R., Ensslen, F., Belis, J.: Size effect model for the edge strength of glass with cut and ground edge finishing. Engineering Structures **79**, 96 -105 (2014)

Veer, F.A., Louter, C., Bos, F.P.: The strength of annealed, heat-strengthened and fully tempered float glass. Fatigue Fract Eng M **32**, 18-25 (2009)

Veer, F.A., Rodichev, Y.M.: The structural strength of glass: Hidden damage. Strength Mater+ **43**, 302-315 (2011)

Weibull, W.: A Statistical Theory of the Strength of Materials. Ingenjörsvetenskapsakademiens handlingar **151** (1939)

Wereszczak, A.A., Ferber, M.K., Musselwhite, W.: Method for Identifying and Mapping Flaw Size Distributions on Glass Surfaces for Predicting Mechanical Response. Int J Appl Glass Sci **5**, 16-21 (2014)

Wiederhorn, S.M., Bolz, L.H.: Stress corrosion and static fatigue of glass. J. Am. Ceram. Soc. **53**, 543-548 (1970)

Yankelevsky, D.Z.: Strength prediction of annealed glass plates -- A new model. Eng Struct **79**, 244-255 (2014)