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Probability Distributions in the Glass Failure Prediction Model

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Glass, a brittle material, fractures under tensile stress acting over a time duration. Lateral loads, such as wind, acting on a simply supported rectangular glass lite, put one surface of the lite primarily into tension. ASTM E 1300 defines load resistance of glass as the uniform lateral loading acting over a duration of 3 seconds that is associated with a probability of breakage of 8 lites per 1000 at the first occurrence of the loading. To determine load resistance, the underlying window glass failure prediction model facilitates determination of a probability distribution of 3 second equivalent failure loads, P_3. The glass failure prediction model is based on a Weibull distribution, and most people believe the distribution of P_3 is, in fact, a Weibull distribution. However, the authors contend that this is not the case. This paper provides an explanation of the glass failure prediction model, its basis, and a discussion of the method for determining surface flaw parameters with an example. The authors demonstrate the distribution of the equivalent failure loads does not follow a Weibull distribution, and they will elucidate the relationship between the distribution of P_3 and the Weibull distribution.

Keywords: Glass Failure Prediction Model, Surface Flaw Parameters, Weibull Distribution, Equivalent Failure Load

1. Introduction

Griffith (1920) showed brittle materials, e.g. glass, fracture at relativity low nominal tensile stress magnitudes due to the presence of surface flaws. Surface flaws are randomly distributed and oriented across the glass surface, thus the location of the flaw that initiates fracture (critical flaw) is unknowable until after fracture has occurred (Griffith, 1920). The location of the critical flaw rarely, if ever, coincides with the point of maximum stress on a glass lite's surface (Natividad, et. al., 2016). Consequently, design procedures based on maximum stress are insufficiently sensitive to produce a consistent level of risk, i.e. probability of breakage. Freshly manufactured glass lites have surface flaw distributions. These surface flaw distributions continually change, due to handling, installation, weathering from environmental exposure, and normal loading conditions. Charles (1958a, 1958b) showed the surface flaw tip radius reduces with an applied tensile stress on the surface acting in combination with water vapor. The reduced flaw tip radius leads to higher stress concentrations at the surface flaw tip. Eventually, when tensile stress concentrations in the neighborhood of a flaw exceed some critical value, fracture initiates at that flaw.

Standard practice ASTM E1300 (ASTM, 2016) provides methodologies for determining load resistance of window glass and window glass constructions. ASTM E1300 defines the glass load resistance as a 3-second duration uniform lateral load associated, with a probability of breakage of 8 lites per 1000 at the first occurrence of the design load. ASTM E1300 finds its basis in the glass failure prediction model (GFPM) advanced by Beason and Morgan (1984). The GFPM accounts for all known factors affecting the strength of glass, including: surface area, aspect ratio, geometry, stress magnitude and orientation, and applied load duration. While the GFPM derives from a Weibull distribution, it is a common misconception that the resulting distributions of glass lite load resistance follow a Weibull distribution. This paper provides a brief review of the Weibull distribution and the GFPM and elucidates the relationship between them. What does and does not follow a Weibull distribution in the GFPM formulation will be explained.

2. Probability Distributions for Brittle Materials

2.1. The Weibull Distribution

Weibull (1939) recognized that brittle materials have a poorly defined and, usually, highly variable ultimate strength due to the environment in which fracture initiates. Therefore, he advanced a probabilistic model to describe the strength of brittle materials. Eq. (1) shows the Weibull cumulative distribution function for isotropic brittle materials:

$$P_B = 1 - e^{-B} \tag{1}$$

where P_B denotes the probability of brittle fracture (breakage, for glass), e denotes Euler's number, and B denotes a risk function.

Weibull (1939) advanced the following risk function to characterize the strength of brittle materials:

$$B = \left(\frac{\sigma}{\lambda}\right)^{\gamma} \tag{2}$$

Where σ denotes the tensile stress at fracture, λ denotes the scale parameter, and γ denotes the shape parameter. This risk function agreed well with empirical strength data for brittle materials. Combining the risk function, Eq. (2), with Eq. (1), produces the common two parameter form of the Weibull cumulative distribution function shown in Eq. (3):

$$P_B = 1 - e^{-\left[\left(\frac{\sigma}{\lambda}\right)^{\gamma}\right]} \tag{3}$$

Weibull's risk function addresses only the stress at failure and none of the other factors known to affect the strength of glass. The GFPM accounts for these other factors.

2.2. Glass Failure Prediction Model

The GFPM begins with the formulation in Eq. (1) but uses a different form of the risk function. Consider a time varying uniform load P(t) that initiates fracture after acting over a time period t_f. This time varying loading creates time varying maximum principal stresses, $\sigma(t)$, over the glass lite surface. Eq. (4) illustrates the conversion of $\sigma(t)$ at the fracture to the equivalent constant maximum principal tensile stress that would have initiated fracture if it acted over time duration t_d:

$$\tilde{\sigma}_{t_d} = \left\{ \frac{\int_0^{t_f} \left[\sigma(t)\right]^n dt}{t_d} \right\}^{1/n} \tag{4}$$

where $\sigma(t)$ denotes the time varying maximum principal stress at the fracture origin at time t, n denotes the static fatigue constant for glass (taken as 16 herein), and t_f denotes the time when fracture occurs. Temperature and relative humidity are assumed to remain constant over t_d in Eq. (4). A non-linear numerical model, finite element or finite difference, is used to compute the maximum and minimum principal stresses at discrete points over the surface area of the glass plate. Since glass lite fracture rarely if ever occurs at the location of the largest maximum principal stress on the glass lite, $\sigma(t)$, the time varying maximum principal stress, is computed at the fracture origin. Also, a single value of P_{t_d}, the equivalent failure load, corresponds to the value of $\tilde{\sigma}_{t_d}$ and can be determined from the relationship $\sigma(t)$ versus $P(t)$ at the fracture origin.

Looking at an entire lite, the calculation illustrated in Eq. (4) is carried out at all discrete points on the tensile surface of the loaded glass lite. Then at each point (x_i, y_j) the discrete area around the point is converted to an equivalent area:

$$\tilde{A}_{(x_i,y_j)} = A_{(x_i,y_j)} \left[c(x_i, y_j) \tilde{\sigma}_{t_d}(x_i, y_j) \right]^m \tag{5}$$

where $\tilde{A}_{(x_i,y_j)}$ denotes the equivalent area at the location , $\tilde{\sigma}_{t_d}(x_i, y_j)$ represents maximum equivalent principal stress of time duration t_d at location (x_i, y_j) on a glass lite, $A_{(x_i,y_j)}$ denotes the tributary area around the point over which $\tilde{\sigma}_{t_d}(x_i, y_j)$, assumed to be constant, acts, $c(x_i, y_j)$ denotes the biaxial stress correction factor (Beason and Morgan, 1984) at the location(x_i, y_j), and m denotes a surface flaw parameter closely associated with the shape parameter in the Weibull distribution. The total equivalent area, S_m for a glass lite consists of the integral:

$$s_m = \int_0^a \int_0^b \tilde{A}_{(x,y)} dydx \tag{6}$$

where a and b denote the glass lite's rectangular dimensions. The total equivalent area, S_m, provides a measure of damage to a glass lite associated with fracture. The GFPM combines Eq. (4) through Eq. (6) to express the risk function in a manner which incorporates all factors that affect the strength of glass. The risk function (Beason and

Morgan, 1984) characterizes the condition of the surface of the glass lite through two surface flaw parameters, m and k, shown in Eq. (7):

$$B = k\int_0^a \int_0^b \left[c\left(x, y\right)\tilde{\sigma}\left(x, y\right)\right]^m dxdy = ks_m \tag{7}$$

In Eq. 7, k denotes the second surface flaw parameter which is closely associated with the scale parameter in the Weibull distribution. Typically, the surface flaw parameters, m and k, are estimated from fracture data from a sample of rectangular glass lites. In doing so, the best estimate for k is:

$$k = \frac{1}{\bar{s}_m} \tag{8}$$

in which $\bar{s}_m$ denotes the mean of the values computed in Eq. (6) for a sample of glass lites tested to failure. Note that the estimation of m and k requires an iterative procedure to determine the values that provide the best fit.

3. Discussion

As noted above, Weibull (1939) defines the risk function as $\left(\frac{\sigma}{\lambda}\right)^{\gamma}$. If dealing with a situation where σ is a linear function of applied load, P, then the failure loads would follow a Weibull distribution. Comparison of Weibull's risk function with the risk function for the GFPM as shown in Eq. (7) indicates the values of S_m associated with fracture follow a Weibull distribution. But perusal of Eq. (4) through Eq. (7) indicates that the equivalent fracture loads leading to the values of S_m may or may not follow a Weibull distribution.

Two cases come to mind in which equivalent fracture loads follow a Weibull distribution. These cases are four-point bending tests and ring-on-ring tests. In both cases, load, P, is linearly related to maximum principal tensile stress, σ, and the surface area subjected to tensile stress is a constant. In addition to the linear load stress relationship, these two test methods produce special states of stress. A four-point bending test produces uniaxial tension of constant magnitude between the loading points. A ring-on-ring test produces a state of uniform biaxial tensile stress on the glass surface within the loading ring. Consequently, performing either of these tests allows researchers to make estimates of the surface flaw parameters quite easily. But, depending upon the area placed in tension using either of these test methods, the surface flaw parameters may not scale well to full size glass lites. When the area placed in tension is relatively small, a smaller population of flaws exists. Hence, the surface flaw parameters, m and k, estimated from either of these methods using small glass specimens could lead to distributions that overestimate the load resistance of full scale window glass lites. Both states of stress lead to a simple analysis but neither models the true conditions of stress variation in a loaded rectangular glass lite.

On the other hand, if failure strength data are developed by testing full size, simply supported, rectangular lites to fracture under controlled loading, a different scenario arises. To begin, the uniform loading, $P(t)$ is no longer linearly related to maximum tensile stress $\sigma(t)$ at, in general, any point (Abiassi, 1981; Beason and Morgan, 1984; Norville and Minor, 1984). In addition, the biaxial stress correction factor, $c(x, y)$, further impacts the relationship between equivalent uniform load and tensile stress. While Eq. (7) indicates that the values of equivalent area, S_m follow a Weibull distribution for rectangular glass lites, the associated values of equivalent uniform load, P_{t_d}, do not follow a Weibull distribution, regardless of the time duration, t_d, considered.

The beauty of the GFPM lies in the fact that the surface flaw parameters, m and k, depend only on the glass surface conditions and not on the glass lite geometry and other factors. This basic fact allows ASTM E1300 to characterize load resistance for all glass lite geometries in terms of one set of surface flaw parameters.

4. Conclusion

The authors have discussed the mathematics that describe the distribution of glass lite load resistance in some detail. They have further described the conditions under which failure loads can be characterized using a Weibull distribution (glass strength data from four-point bending tests and ring-on-ring tests). Further, they have gone on to make the point that the load resistance for rectangular glass lites, P_{t_d}, associated with a time duration t_d do not follow a Weibull distribution. Instead the equivalent areas, S_m, follow a Weibull distribution. While a one-to-one correspondence exists between P_{t_d} and S_m, that relationship is not linear. Hence, the distributions of load resistance in ASTM E1300 in which $t_d = 3s$ is not a Weibull distribution.

References

Abiassi. J. J., (1981): "The strength of weathered window glass using surface characteristics," *Institute for Disaster Research*, Texas Tech University, Lubbock, Tex., Aug.

ASTM (2016): Standard practice for determining load resistance of glass in buildings. ASTM E 1300-16$^{a\varepsilon1}$, West Conshohocken, PA.

Beason, W. L., and Morgan, J. R., (1984): "Glass failure prediction model," *Journal of Structural Engineering*, 111(9), 2058–2059.

Charles, R.J., (1958a): "Static fatigue of glass, I," *Journal of Applied Physics*, 29(11), 1559-1553.

Charles, R.J., (1958b): "Static fatigue of glass, II," *Journal of Applied Physics*, 29(11), 1554-1560.

Griffith, A. A., (1920): "The phenomena of rupture and flow in solids," *Philosophical Transactions of the Royal Society of London*, Vol. CCXXI-A587, Oct. 163-179.

Natividad, Kayla, Morse, S.M., and Norville, H.S., (2016): "Fracture origins and maximum principal stresses in rectangular glass lites," *Journal of Architectural Engineering, ASCE*, 22(2).

Norville, H.S. and Minor, J.E. (1985): "The strength of weathered window glass*," Bulletin of the American Ceramic Society*, 64(11): 1467-1470.

Weibull. W., (1939): "A statistical theory of the strength of materials," *Ingeniors-vetenskapsakademiens*, Handlingar NR151, Stockholm, Sweden.