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Damping of Glass Structures and Components

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In this paper, the dynamic properties of a glass structures and glass components will be discussed. A short introduction to dynamic load will be presented as well as the basic principles of calculus used behind the widely understood finite element analysis. A comprehensive review of the various damping concepts, coefficients and physical backgrounds will be summarized. The mathematical model is analyzed by Finite Element Method and by STRAND 7 software system. In this paper, the consideration of various damping concepts using FEM for the harmonic and transient dynamic analysis is addressed. Several key points are summarized.

Keywords: Glass, Structure, Time history analysis, Damping

1. Introduction

Glass, as a load bearing material, experienced extensive research & development in past decades. Researchers have focused to establish the safe design properties of the glass and glass composites for short and long term loading periods. These results have become a backbone of the various standards and code requirements used in everyday design. However, even with the enormous effort, some of the basic design parameters are almost unknown and others are still under discussion.

In conjunction with research, more advanced glass structures have been built. Nowadays, even more braver and complex glass structures have been completed. In [4] the authors published a wide range of projects, in which they successfully used glass to a new potential. The development of new, laminating interlayer and techniques, allows the use of glass in an expanding range of structural applications. Comprehensive knowledge about basic material properties is essential to continue the advancement of this exciting structural field.

2. Dynamic Analysis

We are living in a four dimensional world, where everything around us is in constant movement. Complex 3D structures are often approximated to 2D even 1D simplified mathematical model. Considering steady state external loads is also common practice in the everyday structural design. However, if the investigated structure is sensitive to vibration, or applied forces acting in the period near to the natural frequency, loads needs to be considered as variable with time and dynamic analysis has to be adopted. This will allow us to study the behavior of the structure in the time.

Elementary dynamic loads with approximate reaction times are listed below:

- Blast loads, hard and soft body impacts, short term, /milliseconds/
- Wind load, medium term, /seconds/
- Seismic load, medium term, /seconds/
- Human or technological induced vibrations, from short to long term

Advanced time history analysis has spread in structural practices in the past decade. Powerful structural FEA software is able to solve complex structure quickly and in increasing detail. Current modern code requirements also often require and encourage engineers to carry out dynamic analysis to validate the dynamic behavior. As an example, a spectral analysis is well established in all earthquake codes or vibration analysis for human induced vibrations.

As a result of this type of enhanced analysis, it is very sensitive to input parameters and therefore an experienced and competent approach is needed. Damping is the basic material property defining the behavior of the structure subjected to dynamic load.

In our practice, we often face the problem, of how to estimate the suitable damping ratio for glass structures. We researched for the answer through published handbooks and codes of practice dealing with glass design and dynamics and found very little which was of help. It is quite understandable as structures made from structural glass are still uncommon.

Therefore we decided to summarize our thought on this subject in the following paper. Our primary aim was, identify this problem and mobilize the scientific community to further research on this topic.

2.1 Methods of solution

Two general methods for dynamic calculations are known. For simple models with restricted degrees of freedom, an analytical method is very effective and exact. This method can be used for solving and verifying results on simple glass components such as beams or fins. For complex geometrical structures with various boundary conditions an analytical method is not appropriate, so a finite element method is used.

$$[M]\frac{d^2u}{dt^2}+[C]\frac{du}{dt}+[K]u=f_t \tag{1}$$

To solve equation (1) a direct integration method can be used. Time history loadings with various load patterns can be easily calculated by using transient dynamic analysis.

2.2 Mechanical approach

The single degree of freedom (SDOF) is the simplest dynamic model and consists of an oscillator with a concentrated mass. Well known analytical solutions can be written for basic models:

- Motion without damping
- Motion with damping

It is very rare to find civil engineering structures exhibiting anything close to critical damping, therefore in this paper we assumed only under damped structures. This assumption is appropriate as typically the damping ratio is in the range 0.02 to 0.1. As it is not possible to analytically determine the damping coefficient c or damping ratio ξ for particular structure, evaluation of damping from experiments is necessary. 'Damping can be experimentally measured by tracing logarithm of the difference between two consecutive peaks in a displacement versus time plot for free vibrations.[2]' The rate of decay strongly depends on the damping ratio ξ. It can be shown that for system with low damping, the ratio of any two successive peaks is

$$\frac{u_i}{u_{i+1}} \approx e^{2\pi\xi} \tag{2}$$

The logarithmic decrement, δ, is used to find the damping ratio of an under damped system in the time domain. The logarithmic decrement is the natural log of the amplitudes of any two peaks:

$$\delta = \frac{1}{n}\ln\frac{u_0}{u_n} \tag{3}$$

Two successive peaks may have very similar displacement so considering two peaks that are several cycles apart is a better approach, to avoid erroneous results.
The effective damping coefficient for a structure is defined as the damping ratio calculated for entire structural model. For small values of the effective damping coefficient $\xi > 0.1$, this can be calculated according to following formula:

$$\xi \approx \frac{\delta}{2\pi} \approx \frac{c}{c_{cr}} \tag{4}$$

The damping ratio is also given by the ratio of critical damping to real damping.

2.3 Response to harmonic excitation

The theory of steady state response of structures to the harmonic excitation has several applications including the forced vibration tests. If we subject the investigated structure to the external force varying harmonically with the time $f(t) = f_0 \cos \omega_f t$, we will obtain the response of the structure in two parts, the free vibration response decayed in time and the steady state response.

A dimensionless response factor D is equal to the dynamic to the static displacement response amplitude. The response factor D depends only on two parameters:

a) Ratio of the frequency of the external harmonic force to the natural frequency of the structure, $\gamma = \omega_f/\omega$
b) Damping ratio ξ

$$D = \frac{1}{\sqrt{\left(1-\chi^2\right)^2 + \left(2\xi\chi\right)^2}} \approx \frac{u_{dyn}}{u_{stat}} \tag{5}$$

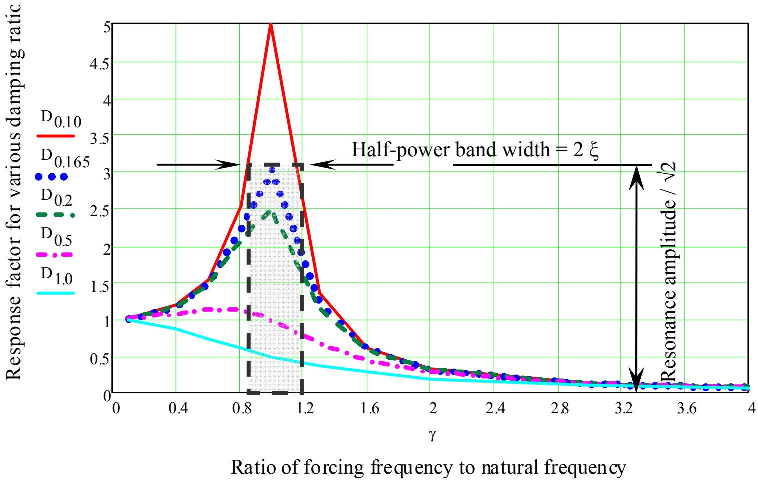

Figure 1: Evaluation of damping from harmonic vibration test.

2.4 2.3 Viscoelastic approach

A very different approach to the mechanical oscillator vibration is the rheological Kelvin - Voigt model in which viscosity is investigated. Damping is a term used for the measure of the energy loss in a dynamic system. Damping dissipates energy and causes the amplitude of free vibration to decay in time. This material damping is referred to as hysteric damping. 'It has been observed from experiments carried out on many materials and structures that under harmonic forcing the stress leads the strain by a constant angle δ [2].'

We can assume that the glass acts as a purely elastic material, while the polymeric interlayer will behave as viscoelastic material. The real part or storage modulus describes the capacity of the material to store energy elastically upon deformation. The imaginary part is called loss modulus since it provides a measure of energy dissipated in the material. Both modules showed a dependence on frequency. The phase shift is delta between stress and stain is given by

$$\eta = \tan\delta = \frac{G''}{G'} \tag{6}$$

Tan delta also known as loss coefficient related to the dissipated mechanical energy ΔU. U_e is stored elastic energy. Loss factor can be therefore written as

$$\eta = \frac{\Delta U}{2\pi U_e} \approx 2\xi \tag{7}$$

An equivalent viscous damping coefficient c_H is given as

$$c_H = \frac{\eta k}{\omega} \tag{8}$$

'Experiments on the damping that occurs in solid materials and structures that have been subjected to cyclic stressing have shown the damping force to be independent of frequency. [6]'. This material damping is known as hysteric damping. Since the viscous damping is depended upon the frequency of oscillation, it is difficult to separate the stiffness of the structure from its hysteric damping.

Table 1: Dynamic properties of Materials, [1], [6]

Material	Loss factor [η]
Glass	$1x10^{-5} – 2x10^{-3}$
Steel	$1x10^{-3} – 8x10^{-3}$
Concrete	$6x10^{-3} – 6x10^{-2}$
Polymer	0.1 - 2

The loss coefficient according to [1] for various common materials is shown in the above Table 1. The loss coefficient for glass is in the range from $2x10^{-4}$ to $1x10^{-5}$, however other authors [2] state higher values from $2x10^{-3}$ to $6x10^{-4}$. Despite the wide range of measurements, it obvious that glass has very low inherent damping in comparison to the other materials.

Fortunately, in present design practice, glass sheets are often laminated with high damping polymer interlayer similarly. Composite materials where stiff fibres are embedded in highly damped matrix material can possess high damping properties. The damping arises from the polymer network after is has been deformed. 'Both frequency and temperature have a large influence on the molecular motion and hence on the damping characteristics [2].' When the composite material vibrates the damping layers are subjected to shear, which cause the energy to be converted to heat and dissipated.
In [2] authors pointed out the consequently significant dependence of the composite beam loss factor on the shear parameter defined as

$$\Gamma = \frac{2l_x G_j}{t_j t_i E_i} \tag{9}$$

For composite beams with rigidly constrained damping layer, where energy is dissipated only though shear strain of the damping layer the approximate loss factor according to [8] can be found as:

$$\eta_s = \frac{2\pi G_j'' l_x^2}{3E_i t_j t_i} \tag{10}$$

3. Damping in the FEA

Damping in finite element method can be introduced by followed basic models [7]:

- Rayleigh damping. - Also known as proportional damping.

$$[C] = a[M] + b[K] \tag{11}$$

- Modal damping. - Modal damping is defined in mode space. It can be assigned for each vibration mode separately.
- Viscous damping. - Viscous damping uses the following expression to calculate the element damping matrix. The global matrix is obtained by assembling all element damping matrices. The element damping matrix is calculated from the viscous damping coefficient *c* assigned to the element properties.

$$[C_e] = \int_{V_e} N^T c.N.dV \tag{12}$$

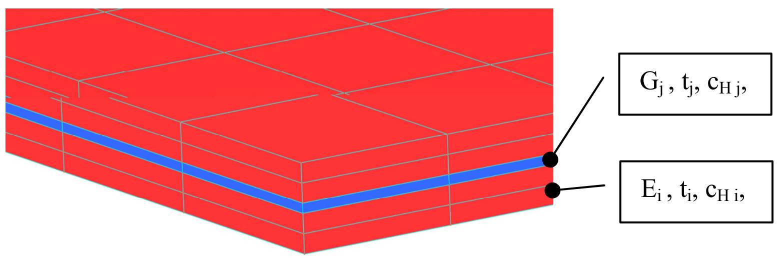

Figure 2: Composite beam, Stiff solid, (glass) E_i, t_i, $c_{H\,i}$, Viscous solid (polymer) G_j, t_j, $c_{H\,j}$, length l_x

4. FEA model

Figure 3 shows the mathematical model of the simply supported composite beam. The glass beam is 3000 mm long and 300 mm wide. The elements used are 8 node brick elements with side length not more than 60 mm and thickness of 3mm. Figure 2 shows a detailed view of the proposed layout.
The applied materials properties can be divided to three distinguished groups according to equation (1) to build particular the matrixes as:

- Stiffness: Elasticity modulus *E*, Poisson ratio μ, element thickness *t*
- Damping: Damping ratio ξ, viscous or hysteric damping *c*
- Mass: specific density γ

Stiffness and mass parameters are well established and typically well known values have been used for both glass and polymers. To estimate accurate damping of the proposed composite beam an equivalent viscous damping according to equation (8) has been calculated. The loss factor for glass and polymer has been taken from literature summarized in Table 1. The equivalent viscous damping of polymer interlayer is several times higher than value used for glass. To obtain the global damping matrix the method described in chapter 3 under has been applied.

The glass beam specimen has been subjected to harmonic load with 25 cycles with various periods. The force from 50 - 200 N has been considered according to particular stiffness of analyzed beam layout and has been applied to the center of the beam. The structural behavior over time has been studied. The time history deflection of the center node for glass beam specimen has been plotted (Figure 4).

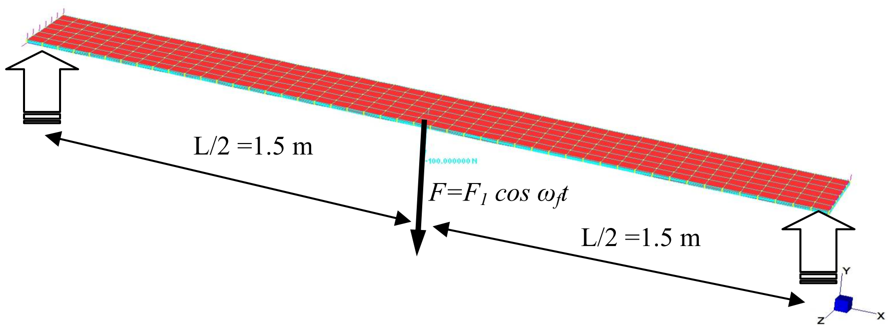

Figure 3: Analysis model of glass beam, 2x6 mm, 1.52 mm PVB, and length = 3m

The linear transient solver was used applied to solve this problem. The full system method (direct integration) based on Newmark method was chosen as this directly and numerically solves the differential equation of (1).

The direct integration method is computationally more intensive as all node displacements are numerically integrated at each time step. The mode superposition method gives more calculation efficiency, however this method is suitable only if low frequencies dominate the response. Another disadvantage of the mode superposition method is that only Rayleigh or modal damping can be adopted. For large MDOF systems subjected to seismic load, an experimentally measured data for modal damping or damping ratio ξ is essential and reduces the time required for analysis.

We adopted direct international analysis technique in very unconventional manner to estimate damping ratio of the proposed glass beam. Input parameters will need to be verified and simple experiments are also required to confirm the introduced theory.

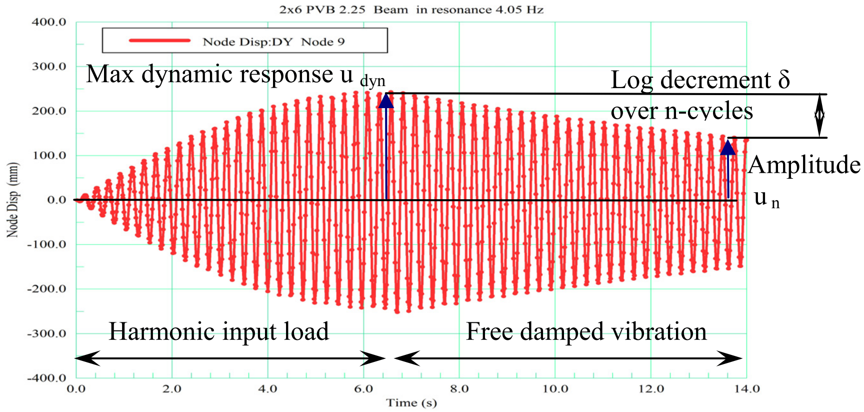

Figure 4: Calculated node displacement over time for composite beam 2x6 PVB 2.25 in resonance 4.05 Hz

Damping ratio has been estimated using well established methods described more in detail in paragraph 2.2 of this paper:

- Free damped vibration evaluation technique The logarithmic decrement has been estimated according to equation (3). Amplitudes have been calculated through the use of the FE model (fig. 4). The damping ratio has been estimated according to the equation (4). The damping ratios for the various composite beam layouts are presented in figure 5.
- Harmonic vibration test evaluation technique. The response factor has been calculated according to equation (5). The dynamic and static amplitudes have been calculated through the use of the FE model (fig. 4). The damping ratio has been estimated with accordance to the half power band width theory (fig. 1), and the calculated response factors over frequency range for various composite beam layouts are presented in figure 6.

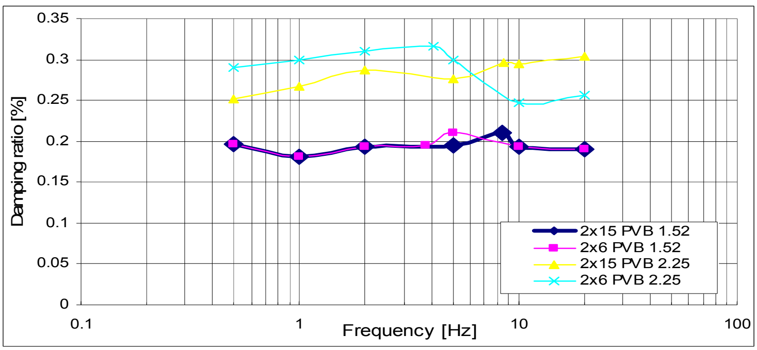

Figure 5: Response to free damped vibration, Damping ratio.

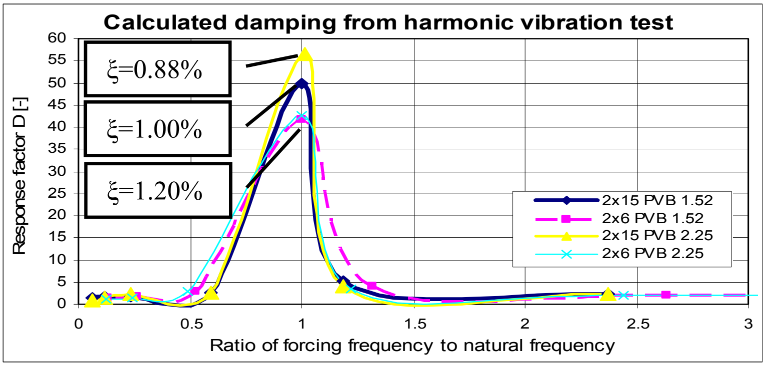

Figure 6: Response to harmonic excitation, Resonance factor.

5. Conclusion

Glass, with strong molecular bonds, does not provide sufficient inherent damping. Improvement in the dynamic behavior of the glass structure and components can be observed, when the laminated composite layouts have been adopted. Due to post failure behavior, non laminated glass elements are very rare these days. Polymers with high damping properties are well known established to the glass industry for decades.

The backgrounds of dynamic analysis have been summarized and two different approaches for damping evaluation have been outlined. The variables which the damping of the composite beam is dependent on have been stated. A short introduction of the various damping models used in finite element analysis has been presented. A simple supported three layered beam has been analyzed with a time history analysis under a harmonic load. The calculated response factor and damping ratio for the various composite beam samples over a frequency range have been presented.

For free damped vibration a fairly constant damping ratio over the all examined models has been calculated in range of 0.2% to 0.3%. From the harmonic excitation analysis the response factor has been plotted (Figure 6). Using the half-power band width method, the damping ratio has been estimated in range of 0.88% to 1.2%. However, damping ratios calculated with both methods do not correlate, therefore further study is required. A simple experimental test of three layered simply supported beam will be essential to gain empirical results. Experimental evidence would help to focus the finite element analysis and verify the results. After that, an extension to multilayered glass component can be made. Nevertheless, investigated glass composite elements proved the well known fact that combining stiff and soft materials a highly damped material can be made. Composites have several times higher damping ratios than equally stiff pure glass sheets. Inherent damping is very low for all types of materials and is not sufficient for real structures, therefore other damping sources need to be outsourced.

Once again an analogy to bolted steel structures can be suggested, where predominantly structural damping results from friction and slip at the connections. This is more significant than the damping inherent in the raw material. In glass structures, this damping is even more valuable as connection details are made up from miscellaneous stiff and viscoelastic materials. Other very significant portion of the damping can by found in non structural connections. Glass is often sealed against moisture with viscoelastic material with high damping parameters. These non-structural joints will also significantly increase the overall damping of the structure.

As pure glass structure becomes more complex, further study for the estimation the dynamic behavior and dynamic properties is more than essential.

6. References

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