Challenging Glass 2 – Conference on Architectural and Structural Applications of Glass,
Bos, Louter, Veer (Eds.), TU Delft, May 2010.

Buckling resistance and buckling curves of pane-like glass columns with monolithic sections of heat strengthened and tempered glass

Markus Feldmann, Katharina Langosch
RWTH Aachen University, Institute for Steel Structures, Germany,
www.stb.rwth-aachen.de

There is some knowledge on the stability behaviour of glass panes under axial loading, i.e. of their buckling stability. However, only a few investigations have been performed so far in this direction, predominantly at cross-sections with mono-panes and sporadically at cross-sections with laminated glass. Thus a research project was initiated by the German Steel Construction Association (DSTV) and the German Ministry of Economics, by which the buckling of pane-like glass columns with mono- and laminated sections was thoroughly investigated. The article focuses on the results for buckling with monolithic sections of heat strengthened and tempered glass, giving buckling curves proposed by derivations and experiments.

Keywords: Glass, buckling curves, experimental test, heat strengthened glass, tempered glass, glass column, glass columns with monolithic sections, design concept

1. Introduction

Transparent architecture demands for transparent load-bearing elements, such as pane-like glass columns of heat strengthened (HSG) or tempered glass (TG). For this laminated section will be is necessary in order to achieve sufficient robustness against impact and also to achieve redundancy, figure 1. Design of such load bearing glass structures necessitates the knowledge about the stability behaviour of laminated glass panes and appropriate technical rules. However, before laminated glass is investigated the buckling behaviour of monolithic glass columns needs to be resolved. Therefore the aim of the analytic and experimental investigations of this paper is to derive simple and consistent design rules for pane-like glass columns with monolithic sections of heat strengthened and tempered glass under axial loading. The proposed design rules are verified by existing buckling tests [1, 2, 3, 4, 5] and by experimental tests and numerical simulations [6] of the RWTH Aachen University.

The presented results are a part of a research project [6], the scope of which was in addition to the monolithic pane-line glass columns also glass columns with laminated glass sections. Insofar the results for the monolithic glass columns are also basis for the research the buckling behaviour of glass columns with laminated sections and their technical rules.

Figure 1: Application of pane-like glass columns: Glaspavillon Rheinbach (D).

2. Consistent buckling curves for monolithic pane-like glass columns

The inhomogeneous differential equation for slender glass columns under an axial compression force N_E using a sinusoidal imperfection $e(x) = e_0 \cdot \sin(\frac{\pi \cdot x}{l})$, figure 2, can be expressed by

$$w''(x) + \frac{N_E}{EI} \cdot w(x) = -\frac{N_E}{EI} e(x) \,. \tag{1}$$

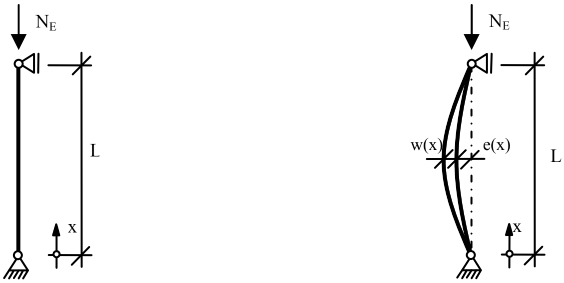


Figure 2: Origin, perfect and deformed imperfect system of a slim column, e(x)=imperfection, w(x)= bending ordinate.

Assuming that bending and imperfection shape are affine, the total deflection in the middle of the column $w_{ges}(x = \frac{l}{2})$ results from both the initial imperfection e_0 and flexural bending deflection w due to the normal force and reads

$$w_{ges} = w + e_0 = e_0 \cdot \frac{1}{1 - \frac{N_E}{N_{cr}}} \,, \tag{2}$$

for which N_{cr} is the Euler buckling force

$$N_{cr} = \frac{\pi^2 \cdot EI}{l_k^{\ 2}} . \tag{3}$$

The stress equation according to 2nd order theory using the magnification factor $\dfrac{1}{1-\dfrac{N_E}{N_{cr}}}$ reads as follows:

$$\sigma = -\frac{N_E}{A} \pm \frac{N_E \cdot e_0}{W} \cdot \frac{1}{1-\dfrac{N_E}{N_{cr}}} . \tag{4}$$

If the value of the imperfection e_0 and the permissible stress f_u are known, the buckling stability can be assessed by equation (4) in the form of a stress verification. However, as the magnitude of the compressive strength of glass differs from that of the tensile strength, the verification of buckling resistance must fulfil both a compression and a tension check:

$$\sigma_t = -\frac{N_E}{A} + \frac{N_E \cdot e_0}{W} \cdot \frac{1}{1-\dfrac{N_E}{N_{cr}}} \le f_{u,t} \text{; and } f_{u,t} = \begin{cases} 70\dfrac{N}{mm^2}\ for\,TG \\ 120\dfrac{N}{mm^2}\ for\,HSG \end{cases} \tag{4a}$$

$$\sigma_c = -\frac{N_E}{A} - \frac{N_E \cdot e_0}{W} \cdot \frac{1}{1-\dfrac{N_E}{N_{cr}}} \le f_{u,c} \text{ and e.g. } f_{u,c} = -500\frac{N}{mm^2} \tag{4b}$$

In view of a consistent verification format, which avoids the double check for both the compression and tensile case, buckling curves are to be proposed for monolithic pane-like glass columns, which are independent of the glass strength but are able to separate the range of the compressive strength from that of the tensile strength. The background for this purpose are buckling curves in the established European form

$$\chi_t = \frac{N_u(\bar{\lambda}_t)}{A \cdot f_{u,t}} , \tag{5}$$

which depends on the non-dimensional slenderness

$$\overline{\lambda}_t = \sqrt{\frac{A \cdot f_{u,t}}{N_{cr}}}\,. \tag{6}$$

Reference value of the strength shall be the standardized tensile strength $f_{u,t}$ (index "t" at $f_{u,t}$, $\overline{\lambda}_t$ and χ_t). The stress equation (4a) then reads using the variables $\overline{\lambda}_t$ and χ_t:

$$0 = -\chi_t + \chi_t^2 \cdot \overline{\lambda}_t^2 + \chi_t \cdot \eta - 1 + \chi_t \cdot \overline{\lambda}_t^2\text{; and } \eta = \frac{e_0 \cdot A}{W} \tag{7}$$

Implementing a parameter $e_0(\overline{\lambda}_t)$ considering the effect of the geometric imperfection of the glass member

$$e_0(\overline{\lambda}_t) = \alpha \cdot \overline{\lambda}_t \cdot \frac{W}{A}, \tag{8}$$

equation (7) can be written in the Ayrton-Perry-format:

$$(1 + \chi_t) \cdot (1 - \chi_t \cdot \overline{\lambda}_t^2) = \chi \cdot \alpha \cdot \overline{\lambda}_t\,. \tag{9}$$

The solution of equation (9) is the function of the buckling curves $\chi_t(\overline{\lambda}_t)$ for that range of slenderness, in which tensile failure is decisive

$$\chi_t = \frac{1}{\phi_t + \sqrt{\phi_t^2 + \overline{\lambda}_t^2}} \tag{10}$$

$$\text{with } \phi_t = \frac{1}{2} \cdot (-1 + \alpha \cdot \overline{\lambda}_t + \overline{\lambda}_t^2)\,. \tag{11}$$

Analogously, but with different sign, equation (4b) describes that range of slenderness, in which the compression failure is decisive

$$\chi_c = \frac{n_f}{\phi_c + \sqrt{\phi_c^2 - n_f \cdot \overline{\lambda}_t^2}} \tag{12}$$

$$\text{with } n_f = \left|\frac{f_{u,c}}{f_{u,t}}\right| = \begin{cases} 7{,}14 \; for\, TG \\ 4{,}17 \; for\, HSG \end{cases};\; \chi_c = \frac{N_u(\overline{\lambda}_t)}{A \cdot f_{u,t}} \tag{13}$$

$$\text{and } \phi_c = \frac{1}{2} \cdot (1 + \alpha \cdot \overline{\lambda}_t + n_f \cdot \overline{\lambda}_t^2) \tag{14}$$

The variable α results from the equation (8) and can be written as:

$$\alpha = \frac{e_0 \cdot A}{W \cdot \overline{\lambda}_t} = \frac{e_0}{l} \cdot \sqrt{3} \cdot \pi \cdot \sqrt{\frac{E}{f_{u,t}}} \tag{15}$$

Using an effective imperfection value e.g. $e_0 = l/400$ (this effective imperfection was verified in [2, 4] and [6] for buckling test with centric normal force), so $\alpha^{TG} = 0{,}430$ and $\alpha^{HTG} = 0{,}329$ yield from equation (15).

As a result figure 3 shows the so derived buckling curves with non-dimensional slenderness relating to tensile strength for heat strengthened and tempered glass. Thereby the range, in which the failure due to reaching the compressive strength or due to reaching the tensile strength is decisive, is visible.

The intersection point of the buckling curves $\chi_t(\overline{\lambda}_t)$ with $\chi_t = 1{,}0$ can be considered as a horizontal curve shift like the European buckling curves for steel columns [7] incorporate. For attaining a formal compatibility with the European buckling curves the buckling curves for glass columns can be written:

$$\chi_t^* = \frac{1}{\phi^* + \sqrt{\phi^{*2} - \overline{\lambda}_t^2}} \text{ with } \overline{\lambda}_t \geq \begin{cases} 0{,}89\,(=\overline{\lambda}_{t,0})\; for\, TG \\ 0{,}92\,(=\overline{\lambda}_{t,0})\; for\, HSG \end{cases} \tag{16}$$

$$\text{and } \phi^* = \frac{1}{2} \cdot (1 + \alpha \cdot (\overline{\lambda}_t - \overline{\lambda}_{t,0}) + \overline{\lambda}_t^2) \tag{17}$$

$$\text{and } \chi_t^* = 1{,}0 \text{ with } \overline{\lambda}_t < \begin{cases} 0{,}89\,(=\overline{\lambda}_{t,0})\; for\, TG \\ 0{,}92\,(=\overline{\lambda}_{t,0})\; for\, HSG \end{cases} \tag{18}$$

Equation (16): $\chi_t^* = f(\overline{\lambda}_t, \overline{\lambda}_{t,0})$ and equation (17): $\phi^* = f(\overline{\lambda}_t, \overline{\lambda}_{t,0})$ are not identical with the equation (10): $\chi_t = f(\overline{\lambda}_t)$ or the equation (11): $\phi_t = f(\overline{\lambda}_t)$ respectively. We see that two different buckling curves depending on the respective glass strength are remaining. Therefore, in order to avoid different buckling curves for heat strengthened and tempered glass the value for α has to be equalized. For this purpose the α -value for tempered glass should be selected also for the heat strengthened glass: $\alpha^{TG} = \alpha^{HSG,new} = 0{,}430$. In this case the effective imperfections are $e_0{}^{TG} = l/400$ and $e_0{}^{HSG} = l/306 \cong l/300$. Thus the proposal for consistent buckling curves in the European form reads (figure 4):

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \overline{\lambda}_t^2}}, \tag{19}$$

$$\phi = \frac{1}{2} \cdot (1 + \alpha \cdot (\overline{\lambda}_t - \overline{\lambda}_{t,0}) + \overline{\lambda}_t^2);\; \overline{\lambda}_{t,0} = 0{,}89;\; \alpha = 0{,}43;\; \chi(\overline{\lambda}_t < 0{,}89) = 1{,}0\,. \tag{20}$$

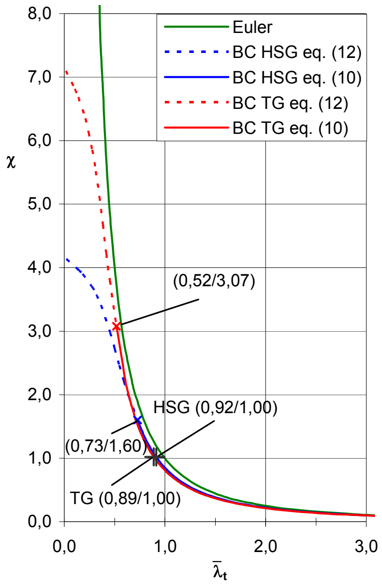


Figure 3: Buckling curves for monolithic glass columns: tempered and heat strengthened glass according to equation (10) and (12).

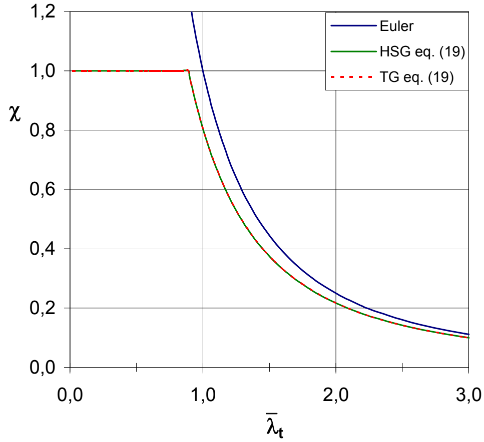


Figure 4: Consistent buckling curves for monolithic glass panes with heat strengthened and tempered glass sections according to equation (19).

3. Experimental tests on monolithic glass columns

3.1. Experimental set-up

The analytic buckling curves have been verified by experimental tests on monolithic pane-like glass columns performed at the RWTH Aachen University. The glass columns were simply supported at its ends according to Euler’s case II. The experimental set-up for buckling and in particular the design of the bearings accord to [2], figure 5. For those hinged bearings at the ends of the glass panes shaft constructions that fit to the groove inside of the bearing roller was provided. In each of the grooves the glass pane was put on a 6 mm block of aluminium and was fastened using adjusting screws, by which a steel mounting plate with an interlayer of Klingersil C4500 was pressed against the glass surface.

The proof load then was applied by a hydraulic jack fixed on the upper bearing and was measured by a load cell. Further the lateral deformation in the middle of the glass pane was measured by a displacement transducer.

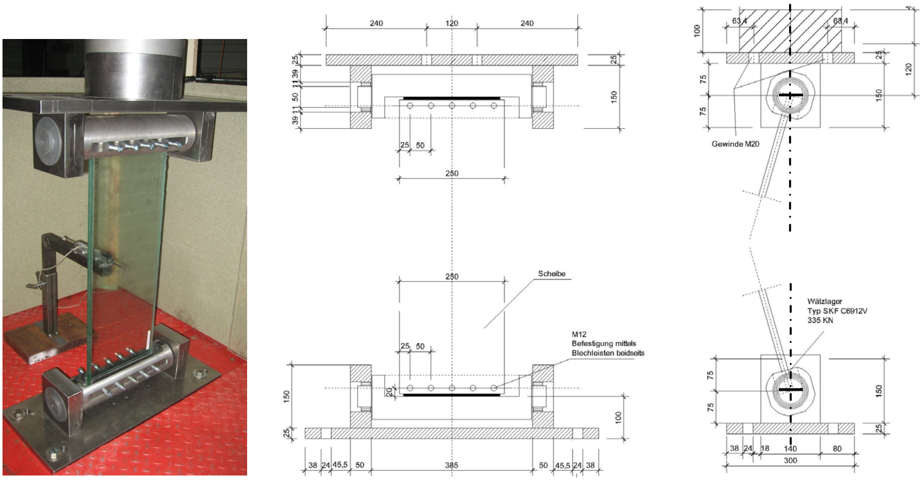


Figure 5: Experimental set-up for buckling of glass-panes according to [2].

3.2. Specimen

A total of 12 specimens (heat strengthened or tempered glass) were tested in buckling, table 1. The dimensions of the specimen were chosen such, that a range of the non-dimensional slenderness of the buckling curves of about $\overline{\lambda} = 1{,}0$ to $\overline{\lambda} = 3{,}0$ was covered. Further the glass columns were tested with centric and excentric normal force.

Table 1: Specimen and test results (remark: buckling length l_k=l+12 mm due to test set up).

Specimen	section dimensions	Span	Tempered Glass (TG) or heat strengthened glass (HSG)	Measured imperfection before buckling test	Regular excentricity	Effective imperfection	Failure load
	W x t	L		e_s	e_p	e_0	F_u
	[mm]	[mm]	[-]	[mm]	[mm]	[mm]	[kN]
1	250 x 10	250	TG	0,05	-	0,53	166,5
2	250 x 10	500	TG	0,10	-	1,94	44,7
3	250 x 10	750	TG	0,40	-	2,08	21,1
4	250 x 12	250	TG	0,00	-	2,97	220,4
5	250 x 12	500	TG	0,10	3,0	6,02	22,3
6	250 x 12	750	TG	0,15	3,0	6,72	22,7
7	250 x 10	250	HSG	0,05	-	0,41	171,2
8	250 x 10	500	HSG	0,05	-	1,41	46,9
9	250 x 10	750	HSG	0,15	-	1,51	21,7
10	250 x 12	250	HSG	0,05	-	0,58	291,9
11	250 x 12	500	HSG	0,15	3,0	6,38	64,3
12	250 x 12	750	HSG	0,20	3,0	7,47	27,3

Before measuring of the thickness, the width, the length and moreover the stresses on the surface of the glass (Scattered Light Polariscope 03) have also exactly been measured and recorded. As a result it could be found that the measurements met the requirements of the EN 1863-1 for tempered glass and the EN 12150-1 for heat strengthened glass.
Further the geometrical imperfections for all available specimens of monolithic or rather laminated glass (84 specimens) was recorded according to EN 1863-1 and EN 12150-1 before the specimens were installed in the set-up. The imperfection of the specimen generally showed a half-sinus wave form, the maximum of which normally was found in the middle of the length. Also those met the requirements of the standard codes.

3.3. Experimental procedure and the results of the buckling tests

The specimens were loaded to failure with a loading rate of 0,10 mm/min at room temperature about 23°C. The breakage of the glass columns always came along with a loud bang. The fracture structure of heat strengthened glass (finest smithereens) was different from that of tempered glass (big pieces), as expected.
The buckling tests were evaluated with the measured, real section dimensions and length. The effective imperfection e_0 (which include all imperfections from the installation the glass columns in the set-up and from the set-up itself) was determined by the so-called "Southwell Plots" [3, 6] and was considered within the numerical and analytical calculations.
The experimental force-displacement-curves and force-stress-curves have been compared to the analytical and numerical calculations, figure 6. The comparison shows well correlation.

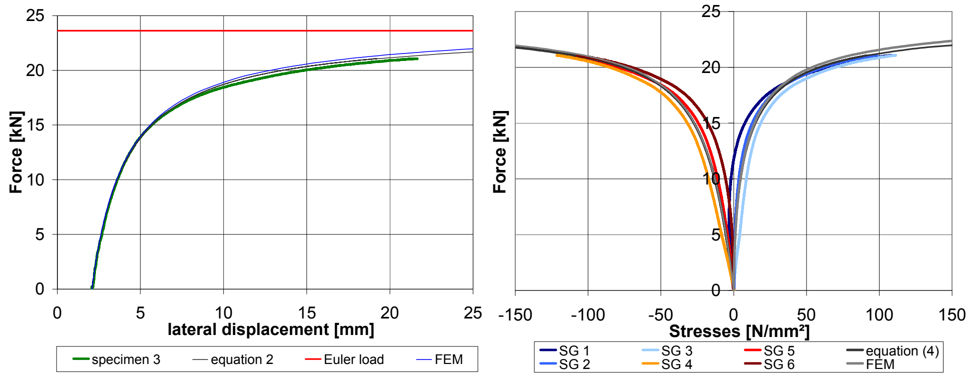


Figure 6: Example: experimental force-displacement-curve (left) and force-stress-curve (right) for the test specimen No. 3 including the analytical and numerical calculations.

Figure 7 illustrates the comparison of all buckling tests being centrically loaded as well as all buckling tests being excentrically loaded. The force-displacement-curves of equal section dimensions and lengths agree each to another except specimen no. 10 and 4. It is also well visible that the buckling failure occurs on a lower load level in case of columns of tempered glass than in case of columns of heat strengthened glass.

Specimen no. 5 with regular excentricity showed a premature collapse, figure 7. The reason for this traces back to the fact that the glass pane showed defects or flaws in the area of the edges. Therefore the results of specimen no. 5 were ignored in the further evaluations.

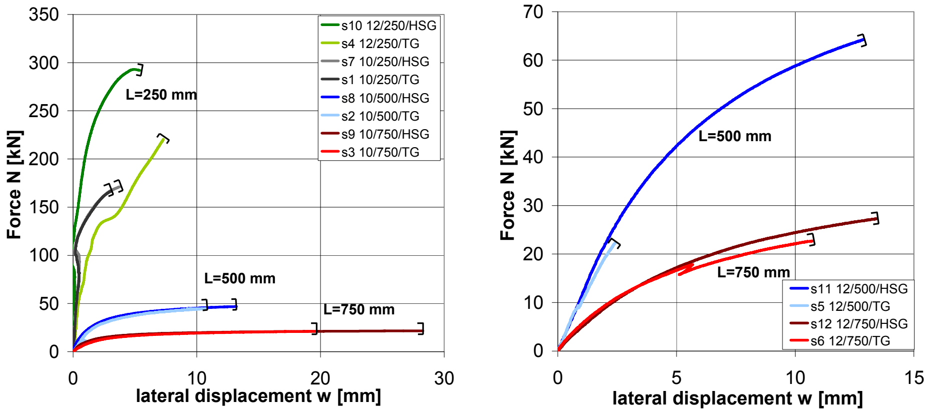


Figure 7: Load-Deformation behaviour of buckling tests at glass panes with monolithic section with centric normal force (left) as well as with normal force and excentricity e_p=3,0 mm (right).

3.4. Comparison of the buckling curves to experimental tests

Figure 8 shows the comparison of all results of specimen without excentricity to the proposal according to equation (10) considering different α -values as well as to the consistent buckling curves according to equation (19) considering uniform α -values.

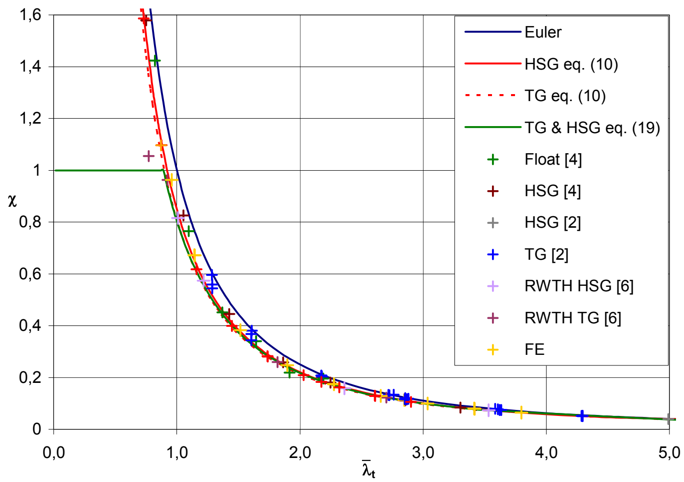


Figure 8: Comparison of the analytic buckling curves for monolithic glass columns to experimental test results without regular excentricity.

Moreover, figure 9 presents the comparison of all experimental buckling results having a regular excentricity e_p (the excentricity was intentionally provided to study the effect of installation tolerances) to equation (10) including an effective imperfection $e_0^{TG} = l/400 + e_p$ for tempered glass respectively $e_0^{HSG} = l/300 + e_p$ for heat strengthened glass. A value representing an installation tolerance is useful and should be considered in the design calculations. The value for this may be (as proposed here) $e_p = 3{,}0\,mm$ being aware that this value need not to be in conformity to any tolerance standards.

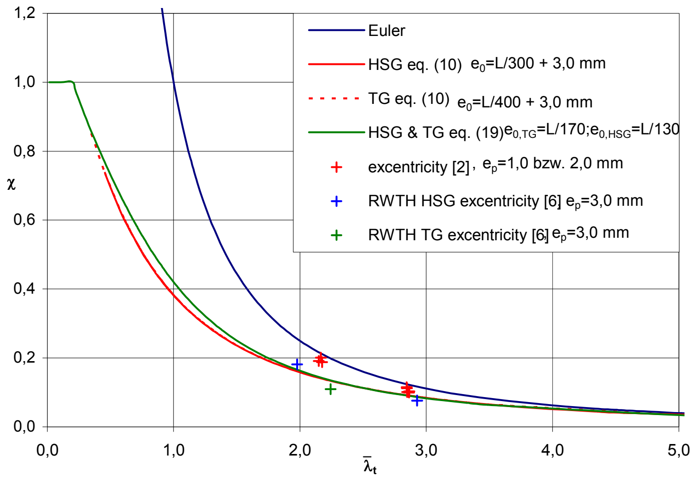


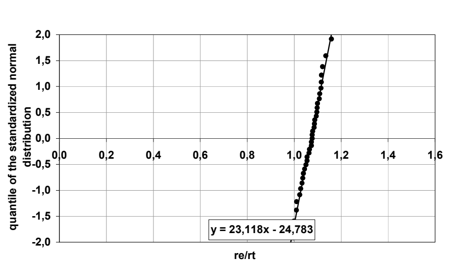
Figure 9: Comparison of the analytic buckling curves for monolithic glass columns to the experimental test results with excentricity e_p.

If the effective imperfections $e_0^{TG} = l/400 + e_p$ or $e_0^{HSG} = l/300 + e_p$ respectively is implemented in equation (10), according to equation (19) the common variables $\alpha_{TVG} = \alpha_{ESG} = 1{,}0$ and $\bar{\lambda}_{t,0} = 0{,}2$ can be determined whilst meeting the format of the European buckling curves. The effective imperfections then can be adjusted in a way so that the values of $\chi(\bar{\lambda})$ consider $e_0^{TG} = l/400 + e_p \cong l/170$ and $e_0^{HSG} = l/300 + e_p \cong l/130$ respectively. For the determination of the partial safety factors γ_M, see chapter 3.5, the corresponding effective imperfections are also taken into an account.

3.5. Determination of the partial safety factor

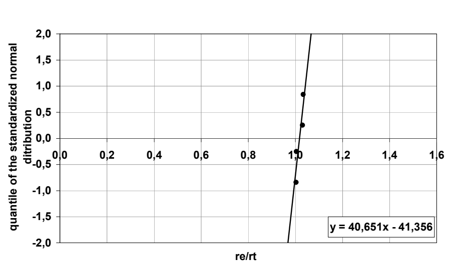
The partial safety factor γ_M was evaluated according to EN 1990 annex D considering 75% confidence probability and a 5% fractile for the characteristic value or rather a 0,1 % fractile for the design value taking into account of real geometries and strengths, figure 10.

TG f_u = 70 N/mm²		
υ_{rt} = 0,15 (geometry, tolerances)		
υ_{fu} = 0,14 (strength)		
standardized normal distribution		



$\bar{b}$ = 1,072	**s_δ = 0,043**	
υ_δ = 0,040 (model)	**υ_R = 0,155 (total)**	
γ_M = 1,412	**Δk = 0,912**	**γ_M^* = 1,287**

HSG f_u = 120 N/mm²		
υ_{rt} = 0,15 (geometry, tolerances)		
υ_{fu} = 0,18 (strength)		
standardized normal distribution		



$\bar{b}$ = 1,017	**s_δ = 0,025**	
υ_δ = 0,024 (model)	**υ_R = 0,152 (total)**	
γ_M = 1,512	**Δk = 0,901**	**γ_M^* = 1,363**

Figure 10: Statistical distribution used for the determination of the partial safety factors for equation (19) considering the imperfection approaches.

The partial safety factors were evaluated for the design formula according to equation (10) or (12) as well as for the formula according to equation (19), separately for both sorts of glass, tempered and heat strengthened glass. As aforementioned, the buckling tests having a regular excenticity were considered, too. The evaluations are shown in table 2.

Table 2: Partial safety factors γ_M .

Design concept	γ_M	
	Tempered glass (TG)	Heat strengthened glass (HSG)
Equation (10) or (12)	1,280	1,390
Equation (19) with e_0=l/400 for TG or rather e_0=l/300 for HSG	1,287	1,363
Equation (19) with e_0=l/170 for TG or rather e_0=l/130 for HSG	1,491	-

4. Numerical Calculations

The monolithic glass columns were modeled by using the FEM simulation software ABAQUS with 3-dimensional C3D8I-elements that provide 8 nodes continuum elements (volume elements). Like in the tests the buckling members were simply supported at its ends according to Euler's case II. On the upper articulation the load was simulated as applied onto the center line of the glass butt end. The material behaviour of glass was implemented by using a modulus of elasticity of E= 70 000 N/mm² and a Poisson ratio of μ =0,23.

For recalculation of the experimental buckling tests the real dimensions and lengths as well as the determined effective imperfection from the experiments were used. A comparison of the numerical simulation to the experiments plus the analytic calculations shows a well correlation of all results, figure 9, and it validates the numerical model for further parametric studies, the results of which are presented in figure 8.

5. Summary

Architectural requirements extend the application of load bearing glass elements, e.g. glass columns and glass beams. With regard to the buckling resistance of pane-like glass columns with monolithic sections of heat strengthened and tempered glass the afore presented investigations can be concluded by the following:

a) On the basis of the second order theory, buckling curves for glass columns are derived from the stress equation, which could be transferred into the format of European buckling curves. The comparison of the proposed analytic buckling curves to experimental buckling tests as well as to the numerical calculations shows a good prediction of the proposed buckling curves, a fact that is also represented by low partial safety factors γ_M.

b) For the effective imperfections the following values are proposed: $e_0 = l/400$ for tempered glass and $e_0 = l/300$ for heat strengthened glass. However, in practice, installation tolerances have always to be considered. These have conservatively been estimated by a value of 3,0 mm for glass columns with a thickness of 12 mm. Considering this, effective imperfection values of $e_0^{TG} = l/400 + 3{,}0\,mm$ or $e_0^{HSG} = l/300 + 3{,}0\,mm$ respectively come out. By this the basis for the implementation of buckling curves as proposed in technical rules or codes are laid down.

c) The research results also provide the foundation for further research projects with laminated glass, e.g. on buckling of glass columns or the lateral torsional buckling of glass beams with laminated glass sections.

The author and the authoress thank the German Steel Construction Association (DSTV) for the sponsorship of the research project with the funding of the German Federation of Industrial Research Associations Otto von Guericke (AiF) of the German Ministry of Economics.

6. References

[1] Güsgen, Joachim, *Bemessung tragender Bauteile aus Glas,* Schriftenreihe Stahlbau - RWTH Aachen, Heft 42, Shaker Verlag, Germany, 1998.

[2] Luible, Andreas, *Stabilität von Tragelementen aus Glas*, Dissertation Thèse N^0 3014, EPFL Lausanne, Switzerland, 2004.

[3] Holberndt, Tobias, *Entwicklung eines Bemessungskonzepts für den Nachweis von stabilitätsgefährdeten Glasträgern unter Biegebeanspruchung*, Dissertation, TU Berlin, Germany, 2006.

[4] Liess, Johannes, *Bemessung druckbelasteter Bauteile aus Glas*, Dissertation, Universität Kassel, Germany, 2001.

[5] Weiler, Hans-Ulrich, *Versuchsergebnisse und Stand der Entwicklung eines Bemessungskonzepts für druckbeanspruchte Glasbauteile*, VDI Berichte Nr. 1527, Bauen mit Glas, VDI Verlag, Düsseldorf, Germany, 2000.

[6] Feldmann, Markus, Langosch, Katharina, *Vereinfachte und einheitliche Stabilitätsnachweise für Bauteile aus Einscheiben- und Verbundsicherheitsglas für Druck und Biegung*, DASt-Forschungsprojekt Nr. 15060/N, Germany, 2009.

[7] Maquoi, R., Rondal, J., *Analytische Formulierung der neuen Europäischen Knickspannungskurven*, Stahlbau 1/1978, Germany, 1978.