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Effect of Asymmetric Heat Transfer on Glass Deformations

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During tempering or other heat treatment processes an uneven temperature field or even an asymmetric temperature field can be created in glass. In an asymmetric case glass can bend depending on the degree of temperature difference. With a known temperature field thermal strains, stress field, and deformations can be calculated. In the paper the theory governing the stress field and deformations is presented. A viscoelastic behavior with the structural relaxation of glass is taken into account.

Keywords: Glass tempering, Bending, Asymmetric temperature field

1. Introduction

In the flat glass tempering process heat transfer should be symmetric around the mid-plane. Simultaneous handling of transient conduction, forced/natural convection, contact heat transfer, and radiation are challenging problems. Due to problems in heat transfer the temperature field can be asymmetric. Depending on the value of the temperature, asymmetry of the temperature field causes bending of flat glass. Deformations of glass depend on the temperature field during the heat treatment process. If the temperature does not rise over the transition temperature only elastic behavior occurs and plastic deformations do not form.

In the elastic case, plasticity does not exist. Thus deformations during heat treatment are easy to simulate [1] and with uniform temperature field glass does not bend. At high temperatures viscosity increases and plasticity is present. The case of a symmetric temperature field has been reported in several articles [2,3,4,5]. In a symmetric case bending does not occur but residual stresses are formed.

In the heat treatment process heat transfer can be asymmetric. For example, forced convection or radiation can cause asymmetrical temperature. The effect of asymmetry has not been reported in the literature. From the standpoint of quality, it would be useful to determine the effect. In the case of glass, viscoelasticity and glass mass should be taken into account. Usually the bending phenomenon should be avoided.

The aim of this paper is to present a theory of the deformations and stress distribution of glass with an asymmetric temperature field. With the theory it should be possible to make an economic simulation program to determine deformations and residual stresses of heat-treated flat glass.

2. Theory

2.1. Heat transfer

Heat transfer and the temperature field form the basis of the heat treatment process. Once the compound of glass has been selected, only heat transfer and temperature range can affect the results. The calculation of temperature distribution is based on the use of the energy equation

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i}\left(k \frac{\partial T}{\partial x_i} \right) + S \tag{1}$$

where ρ is the density, c is the specific heat capacity, T is the temperature, t is the time, k is the heat conductivity and S is the heat source term for radiation.

In order to solve Eq. (1), boundary conditions have to be fixed, which can be easily achieved using the heat transfer coefficient

$$q = -k \frac{\partial T}{\partial x_i} = h\left(T_s - T_\infty\right) \tag{2}$$

Now the next step is to evaluate heat transfer coefficients, which poses a very challenging problem. More detailed information about forced convection and heat source term can be found in the literature [6, 7].

2.2. Stress and strain

In the heat treatment process a stress profile is formed. Because the thickness of the plate glass is significantly smaller than its length and width, the stresses can be assumed to fulfil a one-dimensional plane stress condition. If the effect of gravitation is ignored, the stresses and strains are [1,3]

$$\sigma_x\left(x, y, z, t\right) = \sigma_y\left(x, y, z, t\right) = \sigma\left(z, t\right),\ \sigma_z\left(x, y, z, t\right) = 0 \tag{3}$$

$$\varepsilon_i = \varepsilon_i^{th} + \varepsilon_i^{ve} \qquad \text{(i = x, y or z)} \tag{4}$$

$$\varepsilon_i^{th}\left(x, y, z, t\right) = \varepsilon^{th}\left(z, t\right) = \alpha(z, t)\Delta T\left(z, t\right) \tag{5}$$

$$\varepsilon_x^{ve}\left(x, y, z, t\right) = \varepsilon_y^{ve}\left(x, y, z, t\right) = \varepsilon^{ve}\left(z, t\right),\ \varepsilon_z^{ve}\left(x, y, z, t\right) \neq 0 \tag{6}$$

$$\varepsilon_x\left(x, y, z, t\right) = \varepsilon_y\left(x, y, z, t\right) = \varepsilon^0\left(t\right) + \kappa\left(t\right)z,\ \varepsilon_z\left(x, y, z, t\right) \neq 0 \tag{7}$$

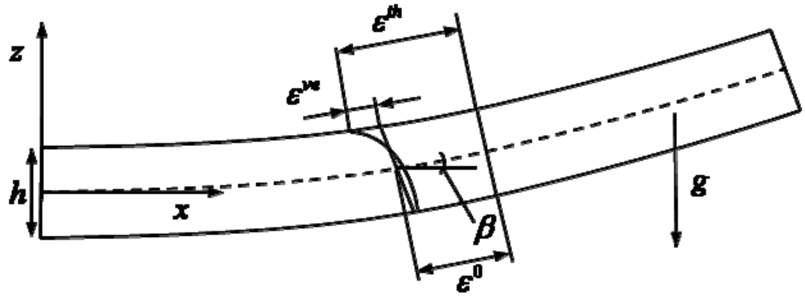

Figure 1: Strains in glass.

In Eqs. (3) to (7), σ and ε are normal stress and strain. The superscript 'th' refers to a thermal and 've' to a viscoelastic effect. Shear stress and strains are assumed to have vanished. Thermal strain depends on the change in temperature and thermal expansion coefficient α. The total strain is composed of strain in the mid-plane ε^0 and the curvature κ. Curvature is the derivative of β

$$\kappa = \frac{\partial \beta}{\partial x} \tag{8}$$

Equilibrium conditions for the force and moment should be valid [1]

$$\int_{-h/2}^{h/2} \sigma(z,t)dz = N \tag{9}$$

$$\int_{-h/2}^{h/2} \sigma(z,t)zdz = M \tag{10}$$

when external forces and gravitation are ignored, N and M are zero.

The relationship between viscoelastic strain ε^{ve} and stress σ can be given with the relaxation function *G(t)* as

$$\sigma = \int_0^t G(t-t')\frac{\partial \varepsilon^{ve}}{\partial t'}dt' \tag{11}$$

The glass relaxation can be described using the Maxwell model, in which a spring and a dashpot are in series. In the model a spring represents solid behavior and a dashpot represents viscous behavior as follows

$$G(t) = G_\infty + (G_0 - G_\infty)\sum_{i=1}^{n} w_i \exp\left(-\frac{t}{\tau_i}\right) \tag{12}$$

Reference values for the material properties and their temperature-dependency can be found in the literature [4,5]. Relaxation values are presented in various ways depending on the reference, either with the bulk and shear moduli or with the auxiliary modulus.

2.3. Structural relaxation

Structural relaxation takes into account the deviation of glass from its equilibrium state. Structural relaxation can be introduced by the change of property due to a change in temperature. The response function $M_p(t)$ of any property $p(t)$ is presented as

$$M_p(t) = \frac{p(t) - p_2(\infty)}{p_2(0) - p_2(\infty)} = \frac{T_f(t) - T_2}{T_1 - T_2} \tag{13}$$

The fictive temperature T_f can be calculated from Eq. (13). In the calculation of the fictive temperature the entire thermal history must be considered

$$T_f(t) = T(t) - \int_0^t M_p(t - t')\frac{dT(t)}{dt'}dt' \tag{14}$$

The response function $M_p(t)$ can be described by the analogy to viscous relaxation as

$$M_p(t) = \sum_{i=1}^{n} C_i \exp\left(-\frac{t}{\lambda_i}\right) \tag{15}$$

The structural relaxation time λ_i is temperature-dependent and its behavior is similar to the stress relaxation time in Eq. (12). When glass is not stabilized, the shift function of the relaxation time depends on the actual temperature and the fictive temperature

$$\ln\Phi(t) = \frac{H}{R}\left(\frac{1}{T_{ref}} - \frac{x}{T(t)} - \frac{(1-x)}{T_f(t)}\right) \tag{16}$$

The shift function (16) should be used during tempering both for stress and structural relaxation. In Eq. (16) the term x is a constant which is dependent on the material.

Reduced time can be calculated with the shift function

$$\xi(t)=\int_0^t \Phi(t')dt' \tag{17}$$

The thermal expansion coefficient is dependent on temperature. At high temperature glass is in the liquid state and the thermal expansion coefficient α_l is about three times greater than at low temperatures when glass is in the solid state α_s. Thermal strain is dependent on the real temperature and the fictive temperature.

$$\varepsilon^{th}(t)=\alpha_l\left(T_f(t)-T_0\right)+\alpha_s\left(T(t)-T_f(t)\right) \tag{18}$$

3. Numerical simulation

3.1. Heat transfer

The temperature field is the basis of the structural simulation and it can be obtained easily by using a home-made implicit difference method [8].The method calculates the temperature distribution and also takes into account the change of material properties (conductivity and specific heat).

3.2. Fictive temperature T_f and reduced time ξ

The fictive temperature is calculated modifying Eqs. (13) – (15) [9]

$$T_{fi}(t)=\frac{\lambda_i T_{fi}(t-\Delta t)+\Delta t T(t)\Phi(t)}{\lambda_i+\Delta t\Phi(t)} \tag{19}$$

$$T_f(t)=\sum_{i=1}^{n} C_i T_{fi}(t) \tag{20}$$

Because the fictive temperature as a function of time t is unknown in the calculation, the shift function has been calculated explicitly

$$\ln\Phi(t)=\frac{H}{R}\left(\frac{1}{T_{ref}}-\frac{x}{T(t)}-\frac{(1-x)}{T_f(t-\Delta t)}\right) \tag{21}$$

3.3. Strain and stress

Eq. (11) shows the response function between viscoelastic strain and stress. The relaxation function *G(t)* can be presented using the reduced time ξ as

$$G(\xi(t))=G_\infty+(G_0-G_\infty)\sum_{i=1}^{n} w_i \exp\left(-\frac{\xi(t)}{\tau_i}\right) \tag{22}$$

Viscoelastic strain can be calculated using the total strain and the thermal strain. The response function is then

$$\sigma(t) = \int_0^t G(\xi(t) - \xi(t')) \frac{d\left(\varepsilon(t') - \varepsilon^{th}(t')\right)}{dt'} dt' \tag{23}$$

The integral above can be split into two parts

$$\sigma(z, t_n) = \begin{aligned} &\int_0^{t_{n-1}} G(\xi(z,t) - \xi(z,t')) \frac{d\left(\varepsilon(z,t') - \varepsilon^{th}(z,t')\right)}{dt'} dt' + \\ &\int_{t_{n-1}}^{t_n} G(\xi(z,t) - \xi(z,t')) \frac{d\left(\varepsilon(z,t') - \varepsilon^{th}(z,t')\right)}{dt'} dt' \end{aligned} \tag{24}$$

$$\sigma(z, t_n) = \begin{aligned} &\frac{G\left(\xi(z,t_n) - \xi(z,t_{n-1})\right)}{G_\infty} \sigma(z, t_{n-1}) + \\ &\int_{t_{n-1}}^{t_n} G(\xi(z,t) - \xi(z,t')) \frac{d\left(\varepsilon(z,t') - \varepsilon^{th}(z,t')\right)}{dt'} dt' \end{aligned} \tag{25}$$

$$\sigma(z, t_n) = \begin{aligned} &\frac{G\left(\xi(z,t_n) - \xi(z,t_{n-1})\right)}{G_\infty} \sigma(z, t_{n-1}) + \\ &\frac{\left(\varepsilon^0(t_n) + \kappa(t_n)z - \varepsilon^0(t_{n-1}) - \kappa(t_{n-1})z - \varepsilon^{th}(t_n) + \varepsilon^{th}(t_{n-1})\right)}{t_n - t_{n-1}} \cdot \\ &\int_{t_{n-1}}^{t_n} G(\xi(z,t) - \xi(z,t')) dt' \end{aligned} \tag{26}$$

By using Eqs. (9) and (10), the unknown parameters $\varepsilon^0(t_n)$ and $\kappa(t_n)$ can be found. When the mid-plane strain ε^0 and the curvature κ are known, displacements can be calculated as follows

$$u(x, z, t) = \int_0^x \left(\varepsilon^0(t) + \kappa(t)z\right) dx \tag{27}$$

$$w(x, z, t) = \kappa(t)x^2 + \int_0^z \varepsilon_z dz \tag{28}$$

4. Conclusions

The control of heat transfer in all glass treatment processes at high temperatures is of vital importance. Residual stresses and displacements are dependent on heat transfer and how glass is supported. With a home-made 1D-program the heat treatment process can be simulated and the explanation for glass behavior can be found without the need of large commercial finite-element codes.

The results of our own program have been compared to those of commercial codes. The reasons for making our own code is that it is cheap, easy to use and very rapid in industrial surroundings compared to commercial finite-element codes.

5. References

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