

Linear Calculation Methods of Composite Panels

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Innovative glazing, which combines polycarbonate and thin glass to composite panels, ensures a slighter alternative to glass composite with a high resistance against manual attack. Based on experimental studies, these thin glass-polycarbonate composite panels are classified as laminated safety glass and security glazing. The study describes analytical models to analyse their structural behaviour for static short time loads. In accordance to the geometrical boundary conditions of the four-point bending test, the composite panel can be described by the beam theory. A multi-layered system is calculated with the sandwich beam theory with a bending and shear deformation. Additionally, an extended approximate solution based on Wölfel is compared to the classical theory of sandwich elements as well as with experimental results.

Keywords: Thin glass, Polycarbonate, Composite panel, Sandwich theory

Symbols

Symbol	Description	Symbol	Description	Symbol	Description
L	Distance between supporting rollers	α	Ratio L_a / L	A_{GTPU}	Shear stiffness of interlayer
L_b	Distance between bending rollers	F	Load	E_g	Young's Modulus of glass
L_a	Distance between supporting roller and bending roller	w_b	Bending deformation	E_{PC}	Young's Modulus of polycarbonate
t_g	Glass thickness	w_s	Shear deformation	E_{TPU}	Young's Modulus of interlayer
t_{TPU}	Interlayer thickness	A_1	Area of glass pane ($b \cdot t_g$)	μ_g	Poisson's ratio of glass
t_{PC}	Polycarbonate thickness	B	Total bending stiffness	μ_{PC}	Poisson's ratio of polycarbonate
d_g	Distance between local glass axis	B_g	Bending stiffness of glass panes about their own axis	μ_{TPU}	Poisson's ratio of interlayer
d_i	Distance between local interlayer axis	B_s	Bending stiffness of glass panes about the centroid axis	k	Coefficient for shear and bending deformation which varies with load and supporting
b	Width of the specimen	B_{PC}	Bending stiffness of polycarbonate about its own axis	M_s	Bending moment of the glass panes due to their distance

1. Introduction

Facade elements, windows or roofs are basically made of glass. But a single monolithic glass pane does not fulfil entirely static requirements and building engineering physics. In many cases a combination of several glass panes as laminated safety glass is required. At the same time, by increasing the demands of the facade, the nominal thickness and the death load increase as well. A composite panel of thin glass and polycarbonate as a laminated safety glass has a favourable effect of these properties. In addition, properties of the building envelope such as thermal insulation and safety are improved (Weimar 2018a, Weimar 2018b, Weimar 2018c).

Generally, sandwich panels consist of three layers and are typically built with a thick low-density core made of rigid foam covered on each side by a thin metal skin. In glass structures, sandwich panels are given by laminated glass and primarily used for passive safety requirements. Especially single glass panes cannot serve a sufficient post-breakage behaviour. By laminating two glass panes, the interlayer adheres to the glass fragments and the sandwich achieves a post-breakage capacity. Nevertheless, the interlayer leads to a different structural behaviour because of a partial shear transfer. The laminate works like a sandwich element. Instead of common sandwich panels, laminated glass obtains two thick faces and an inner soft and thin core. The extension to more glass layers is possible. At the same time, this introduces the innovative thin glass-polycarbonate composite panel as a five layered multi-system sandwich element. The structural behaviour of the laminate is tested with the four-point bending test according to EN 1288-3 (2000). Due to the constant bending without transverse force between the bending rollers, the test is suitable for this

experimental research. Furthermore, the well-known calculation methods for sandwich elements are transferable and adaptable to thin glass-polycarbonate composite panels.

This paper focuses on the extension of the classical sandwich beam theory for an unfractured five-layered laminate. Two different calculation approaches are discussed. Finally, the analytic methods are implemented for thin glass-polycarbonate composite panels and compared to experimental results of the four-point bending test. Furthermore, specific properties of the innovative laminate and the application complement the study.

2. Theory

2.1. General Approaches

Common sandwich constructions consist of two thin, stiff and strong layers separated by a rigid foam core with a low density. The stiffness of the material combination is a result of the distance of the two outer layers. The lamination of the faces with the shear-resistant core result in a high structural behaviour and flexural rigidity. Nevertheless, the structural behaviour is based on the shear transfer of the rigid foam core. By using laminated safety glass, the interlayer is responsible for the shear transfer. Generally, under static load the two limit states »no composite« and »full composite« can be easy to calculate by using analytical or numerical methods. In reality, the shear forces are transmitted partial. Thin glass-polycarbonate composite panels are five layered laminates, which should be analytically calculated by the sandwich beam theory. Furthermore, it is also permitted to use approximate solutions as a simpler calculation method of composite panels with a partial composite. The sandwich beam theory and the approximate solution describes the linear structural behaviour of a multi-layered system. This assumption is given, when the total deformation is lower than the nominal thickness of the laminate. Otherwise, the geometrical linear method leads to higher deformation and stresses than in reality. But a geometrical non-linear calculation is not completely solved with an analytical theory for the four-point bending test. While the ratio of length to width for a beam is defined lower than 0.30, the four-point bending test according to EN 1288-3 (2000) is mostly approximated with a single-span beam loaded symmetrically with two forces vertical to the longitudinal axis. The result is a simpler calculation of the beam. In case of a wide beam, the Young's modulus E of each layer has to be replaced by the term $E/(1-\mu^2)$ with consideration of the lateral strain disability. For all materials a linear-elastic material behaviour without any time dependence during the short time tests is assumed, although polymers show more complex material behaviour.

Figure 1 shows the cross-section of a symmetrical thin glass-polycarbonate composite panel with its denomination in the four-point bending test. The symbols are used in the sandwich beam theory as well as in the approximate solution. A thin glass-polycarbonate composite panel consists of two outer glass panes and at least of one inner polycarbonate sheet laminated by a thermoplastic polyurethane interlayer.

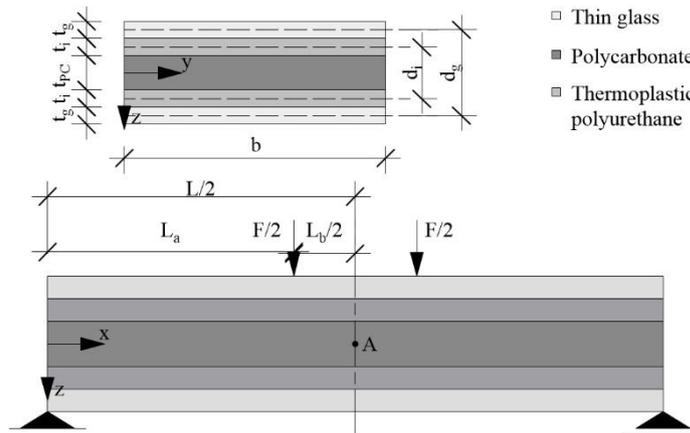


Fig. 1 Geometrical denomination of a symmetrical thin glass-polycarbonate composite panel in the four-point bending test.

Furthermore, the approximation of the local bending stiffness of each layer will be disregarded, if the ratio of the local bending stiffness of the local axis to the bending stiffness of the middle axis is lower than 1 %. The thin face approximation with the unvalued bending stiffness of the faces has to be considered with equation 1 and the weak core approximation with the unvalued core bending stiffness is calculated with equation 2.

$$\frac{2 \cdot B_g}{B_s} < 0.01 \quad \text{if} \quad 3 \cdot \left(\frac{d_g}{t_g} \right)^2 > 100 \quad (1)$$

$$\frac{B_{PC}}{B_s} < 0.01 \quad \text{if} \quad \frac{6 \cdot E_g \cdot t_g \cdot d_g^2}{I - \mu_g^2} > 100 \quad (2)$$

$$\frac{E_{PC} \cdot t_{PC}^3}{I - \mu_{PC}^2}$$

2.2. Sandwich Beam Theory

For every type of structure, the deformations are divided into two parts, one caused by a bending moment and the other by transverse force. Generally, in the classical Euler-Bernoulli beam theory, the shear deformation is completely disregarded due to the fact, that the beam possesses a high shear stiffness like in homogeneous cross-sections. The analysis of multi-layered cross-sections with parts of low shear stiffness is based on the Timoshenko beam theory with the consideration of the shear deformation next to the bending deformation. First of all, the calculation of both deformations shown in figure 2 is done separately and later superimposed.

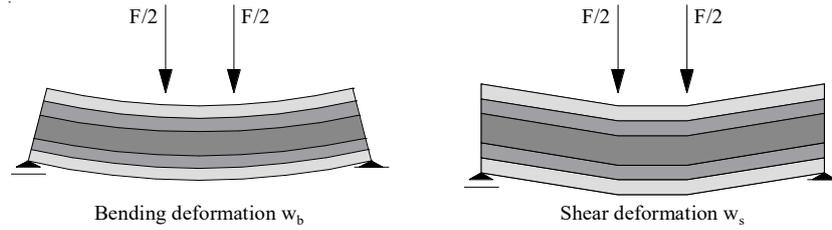


Fig. 2 Bending deformation w_b and shear deformation w_s in a five layered sandwich panel loaded in the four-point bending test.

Due to the extension to a five-layered sandwich beam and due to the regard of the properties of the used material, following simplifications are considered in this study:

- E_g and $E_{PC} \gg E_{TPU}$. The local bending stiffness of the interlayer is disregarded. Furthermore, the assumption of the thermoplastic polyurethane constitutive law has no time dependence due to a short time test. This results in a homogenous and isotropic material.
- The glass faces of the sandwich panel possess a linear elastic material behaviour with a validation of Hooke's law.
- The cross-section is constant, independent of the load and leads to a symmetrical build-up of the laminate.

The flexural rigidity of a cross-section in figure 1 is calculated by equation 3. The first term corresponds to the local bending stiffness of the glass faces about their own axis, the second shows the local bending stiffness of the polycarbonate core about its own axis and the third term represents the stiffness of the faces with bending about the centroid axis of the laminate. Due to the simplifications, the part of the thermoplastic polyurethane interlayer is already disregarded.

$$B = 2 \cdot B_g + B_{PC} + B_s = 2 \cdot \frac{E_g \cdot b \cdot t_g^3}{12 \cdot (1 - \mu_g^2)} + \frac{E_{PC} \cdot b \cdot t_{PC}^3}{12 \cdot (1 - \mu_{PC}^2)} + \frac{E_g \cdot b \cdot t_g \cdot d_g^2}{2 \cdot (1 - \mu_g^2)} \quad (3)$$

The deformation of point A in figure 1 is calculated with equation 4 (Overend 2014). Further information about sandwich constructions can be found in literature according to (Allen 1969) and (Zenkert 1995).

$$w_A = \frac{F \cdot L_A}{2} \cdot \left[\frac{1}{6 \cdot B} \cdot \left(3 \cdot \left(\frac{L_b}{2} \right)^2 + 6 \cdot \left(\frac{L_b}{2} \right) \cdot L_a + 2 \cdot L_a^2 \right) + \frac{1}{A \cdot G_{TPU}} \right] \quad (4)$$

2.3. Approximate Solution

The approximation is based on a solution of (Wölfel 1987) for a sandwich element with elastic shear bond. (Siebert 2012) shows, that the approximation is valid for the calculation of laminated glass. It is similar to the sandwich theory with two stiff layers bonded by a soft interlayer. The model allows a shear deformation between the two stiff cover layers. Hereafter, the method is described for a three-layered system and can be easily extended to a five layered system also with a stiff core. Wölfel considers the elastic bond between the two stiff faces. The shear deformation results of the properties of the shear flexible material. The cross-section consists of two bending flexible but tensile and pressure stiff glass faces with an extensional stiffness. The interlayer possesses no extensional stiffness, but a relevant shear stiffness. The assumptions result in the bending stiffness of the sandwich according to equation (5).

$$B_s = \frac{E_I A_I \cdot E_I A_I \cdot d_g^2}{E_I A_I + E_I A_I} \quad (5)$$

The bending moment M_s can be simply calculated with an analysis of the single-span beam under consideration of the stiffness of the faces. Finally, the deformation in the centre of point A in case of the four-point bending test is determined by equation (6).

$$w = \frac{(1+k) \cdot M_s \cdot L^2}{24 \cdot B_s} \cdot (3 - 4 \cdot \alpha^2) \quad (6)$$

An extension of the approximate solution by Wölfel is possible (Siebert 2012). Based on a typical cross-section with two outer stiff faces and a soft interlayer, the system is modified with an additional interlayer and a new face. Finally, a five-layered system like figure 1 arises with two outer and one inner layer which are laminated by soft interlayers. In the multi-layered system, the bending stiffness of the core material also has an influence on the structural behaviour of the composite panel. Therefore, the core stiffness has to be considered during the calculation as well. The conclusion leads to an extension of Wölfel's approximate solution. The consideration of the bending stiffness of the core will result in an adaption of equation (6).

In comparison of both analytical approaches for the determination of thin glass-polycarbonate composite panels, Wölfel's extended approximate solution shows more conservative results than the sandwich beam theory. Nevertheless, both methods are very sensitive to the input parameter concerning the interlayer's thickness and Young's Modulus shown in figure 3.

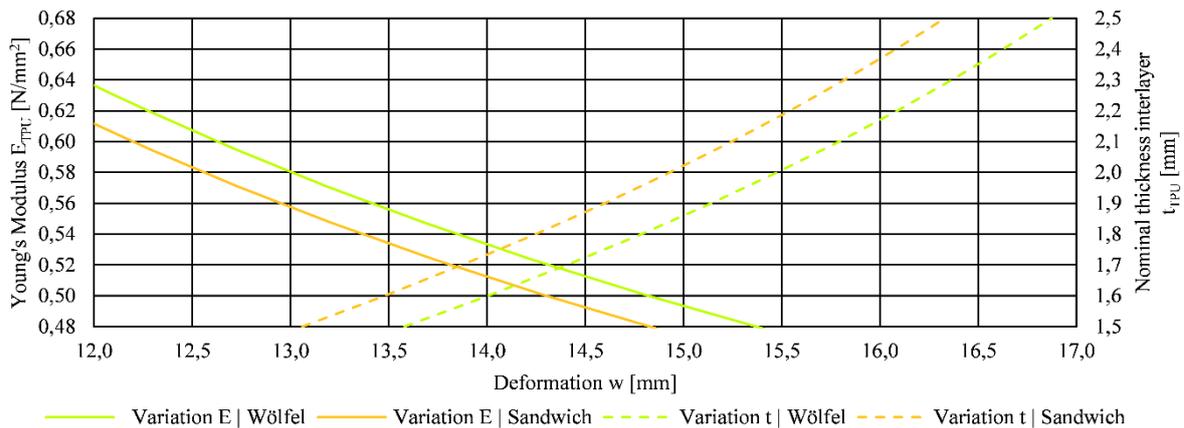


Fig. 3 Comparison between extended theory of the approximate solution by Wölfel and sandwich theory for a thin glass-polycarbonate composite panel analysed in the four-point bending test.

3. Experimental results

The structural behaviour is analysed by the four-point bending test according to EN 1288-3 (2000) as a standard test method for glass. Figure 4 shows the test set-up for a thin glass-polycarbonate composite panel in the four-point bending test with specimen dimensions of 1,100 mm in length and 360 mm in width. The composite panel lays centrally on two supporting rollers and is symmetrically loaded by two bending rollers. Table 1 gives the principle material properties for each layer. The laminate consists of three different materials. The thermal expansion coefficient of polycarbonate is 7 times higher than of glass, so the interlayer with its low Young's Modulus and high thermal expansion coefficient ensures the bonding and shear transfer.

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Table 1: Material properties of a thin glass-polycarbonate composite panel.

Property	Thin glass 01	Thin glass 02	Polycarbonate	Interlayer
Material	Leoflex™ (2015) chemically pre-stressed (CSG)	Optiwhite® (2011) annealed glass (AG)	Lexan® 9030 (2014) (PC)	Thermoplastic Polyurethane (Weimar 2011) (TPU)
Nominal thickness	0.85 mm	2.00 mm	4.00 mm 8.00 mm 12.00 mm	2.00 mm
Density	2.480 g/cm ³	2.480 g/cm ³	2.480 g/cm ³	2.480 g/cm ³
Young's Modulus	74,000 MPa	70,000 MPa	2,300 MPa	0.76 MPa
Poisson's Ratio	0.23	0.23	0.38	0.40
Thermal expansions coefficient	9.8 · 10 ⁻⁶ 1/K	9.0 · 10 ⁻⁶ 1/K	70.0 · 10 ⁻⁶ 1/K	223.5 · 10 ⁻⁶ 1/K

^{*)} The nominal thickness of the inner polycarbonate sheet varies during the experimental studies to analyse the structural behaviour of thin glass-polycarbonate composite panels. Overall, the nominal thickness of the composite panel varies between 9.7 mm and 20.0 mm due to a variation of the polycarbonate sheet and the glass pane. (Weimar, 2018a)

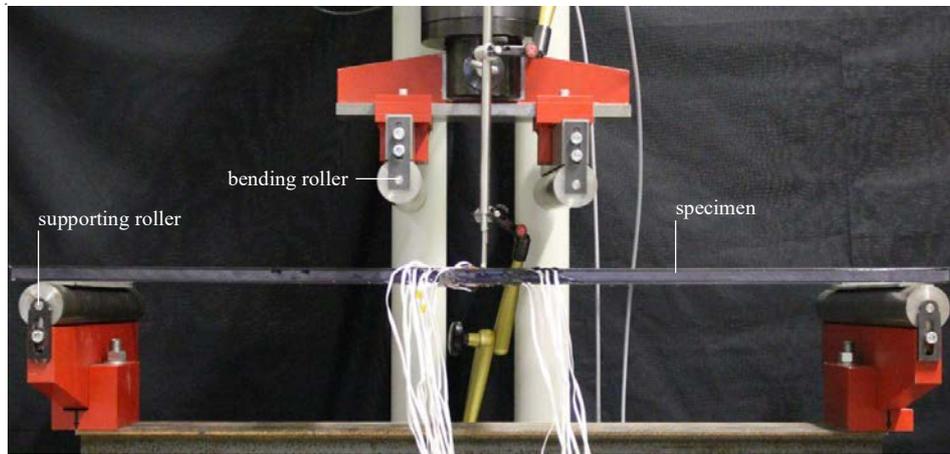


Fig. 4 Test set-up of four-point bending test according to (EN 1288-3) with specimen made of thin glass-polycarbonate composite panel.

The specimens consist of two outer thin glass faces and at least of one inner polycarbonate sheet, which are laminated with a thermoplastic polyurethane interlayer. Overall, three specimens are tested for each cross-section. Table 2 shows the results as mean value for the nominal thickness as well as for the real thickness. Furthermore, the test set-up provides the shear bond of the interlayer, which is a partial composite between the two limit states of no composite and full composite (Weimar 2018a). The tests are realised at a temperature of (+22±1) °C and a humidity of (38±5) % rH. The polymer materials show no time dependence during the short time tests. By increasing the nominal thickness of the polycarbonate sheet, the cross-section is getting stiffer with reduced deformations. Furthermore, the standard deviation decreases due to the more important influence of the sandwich bending stiffness on the total composite bending stiffness.

Table 2: Test results as mean values of the four-point bending test. The bracket term describes the standard deviation.

Specimen	Cross-section	Nominal thickness t_N	Real thickness t_R	Force F	Deformation w	Ratio w / t_R
PK_04_Leo	CSG TPU PC TPU CSG 0.85 mm 2.00 mm 4.00 mm 2.00 mm 0.85 mm	9.7 mm	9.5 mm (< 0.1 mm)	298.7 N (0.3 N)	59.5 mm (8.3 mm)	6.3
PK_08_Leo	CSG TPU PC TPU CSG 0.85 mm 2.00 mm 8.00 mm 2.00 mm 0.85 mm	13.7 mm	13.4 mm (< 0.1 mm)	298.8 N (0.2 N)	25.7 mm (3.5 mm)	1.9
PK_12_Leo	CSG TPU PC TPU CSG 0.85 mm 2.00 mm 12.00 mm 2.00 mm 0.85 mm	17.7 mm	17.3 mm (0.1 mm)	298.6 N (0.3 N)	12.3 mm (0.6 mm)	0.7
PK_04_Opti	AG TPU PC TPU AG 2.00 mm 2.00 mm 4.00 mm 2.00 mm 2.00 mm	12.0 mm	11.7 mm (0.1 mm)	299.2 N (0.3 N)	35.7 mm (3.3 mm)	3.0
PK_08_Opti	AG TPU PC TPU AG 2.00 mm 2.00 mm 8.00 mm 2.00 mm 2.00 mm	16.0 mm	15.5 mm (< 0.1 mm)	299.0 N (0.4 N)	19.3 mm (1.4 mm)	1.2
PK_12_Opti	AG TPU PC TPU AG 2.00 mm 2.00 mm 12.00 mm 2.00 mm 2.00 mm	20.0 mm	19.4 mm (0.1 mm)	299.2 N (0.1 N)	10.8 mm (0.2 mm)	0.6

4. Analytical calculation of thin glass-polycarbonate composite panels

First, the approximation with a thin face and a weak core are analysed. The results of the calculation according to equation 1 and 2 are shown in figure 5 and 6. The bending stiffness about its own axis of the thin glass Leoflex is insignificant for each composite panel. In accordance with Optiwhite glass faces, the bending stiffness about its own axis cannot be disregarded for nominal thicknesses lower than 6 mm. The bending stiffness of the polycarbonate core has to be considered for the composite panels with the thin glass Leoflex higher than a polycarbonate thickness of 8 mm and for laminates with the thin glass Optiwhite higher than a polycarbonate thickness of 5 mm.

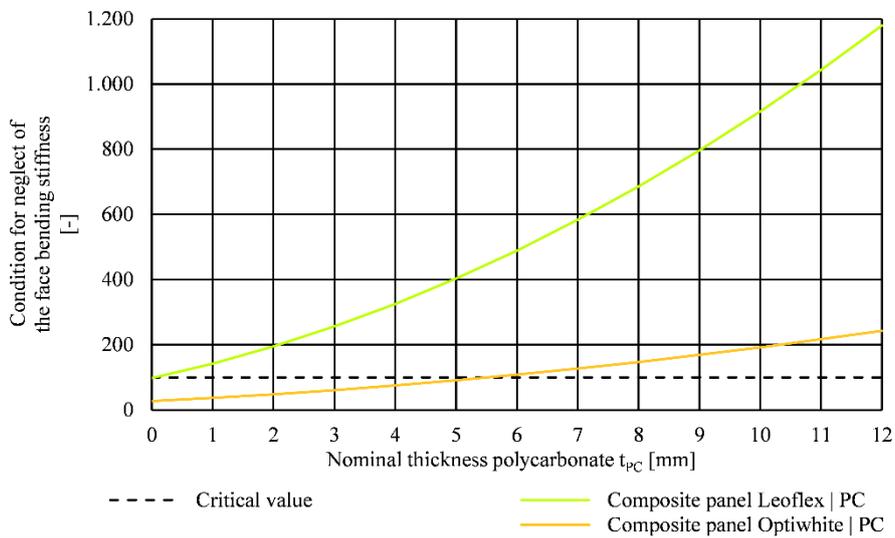


Fig. 5 Thin face approximation for thin glass-polycarbonate composite panels with thin glass Leoflex ($t_g = 0.85$ mm) or thin glass Optiwhite ($t_g = 2.00$ mm) and two polymer interlayers ($t_{TPU} = 2.00$ mm). The nominal thickness of polycarbonate varies.

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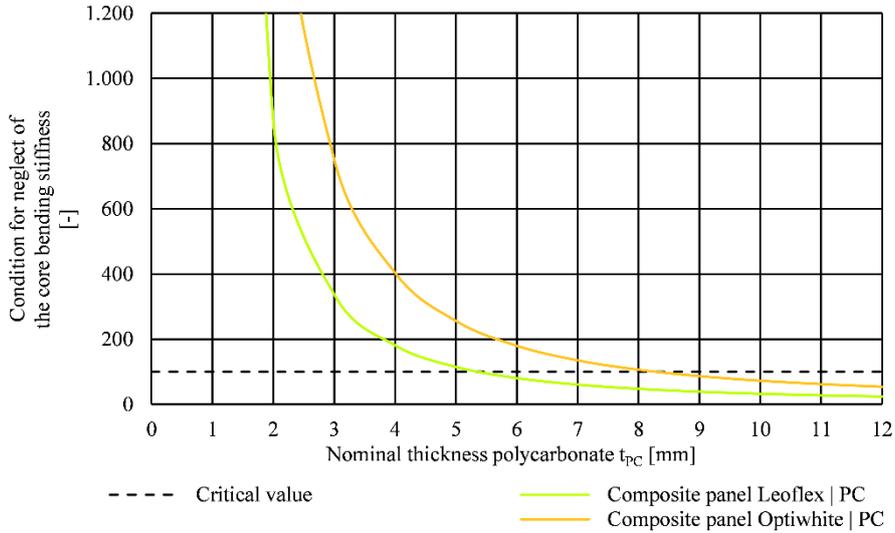


Fig. 6 Weak core approximation for thin glass-polycarbonate composite panels with thin glass Leoflex ($t_g = 0.85$ mm) or thin glass Optiwhite ($t_g = 2.00$ mm) and two polymer interlayers ($t_{TPU} = 2.00$ mm). The nominal thickness of polycarbonate varies.

In accordance with table 2, the linear analyse can only be done with a 12 mm polycarbonate core. For both composite panels PK_12_Leo and PK_12_Opti, the bending stiffness of the faces about their own axis are disregarded, but the bending stiffness of the core has to be considered. Finally, equation (3) is adjusted in regard to the boundary conditions. For the sandwich theory as well as for the approximate extended solution of Wölfel the assumption has to be considered to compare both theories to the experimental study. Figure 7 shows the calculated and measured results in a load-deformation diagram. The material parameter of thin glass and polycarbonate are given in table 1. The Young's modulus of the thermoplastic polyurethane interlayer was calculated by a geometrical non-linear method and a finite element software SJ Mepla 5.0.0. Due to a partial composite in the cross-section, the Young's modulus of each specimen varies. The results show, that the extended theory by Wölfel's approximation almost agrees with the sandwich beam theory. Both calculations lead to conservative results and to higher deformation, which can be followed by the disregard of geometrical non-linear theory and of non-linear material behaviour during the calculation process. By increasing the load, the differences between calculation and experimental values might become more important. Nevertheless, both methods of the sandwich beam theory as well as the extended theory of the approximate solution by Wölfel are useful for the first specification of the deformation under static load.

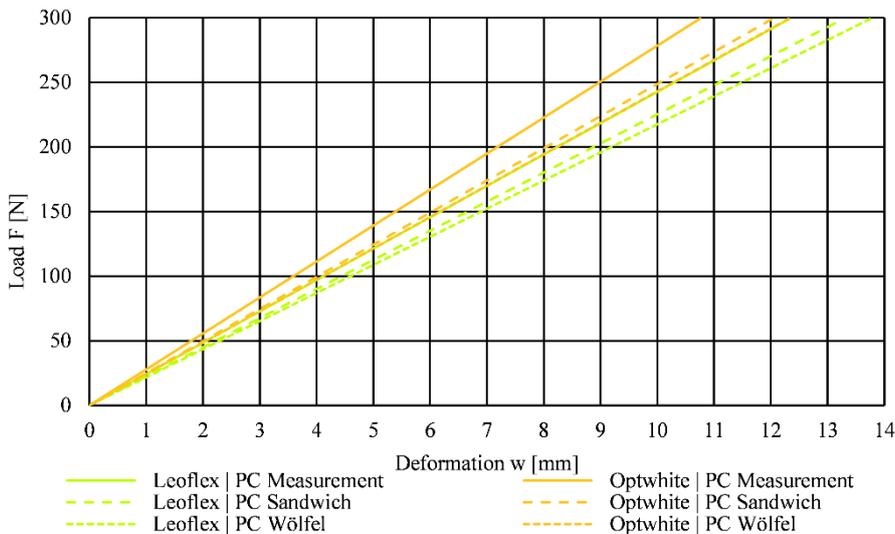


Fig. 7 Comparison of the measured values with the analytical solutions results by sandwich beam theory and extended theory of the approximate solution by Wölfel.

5. Conclusion

The paper described analytical calculation methods for thin glass-polycarbonate composite panels with the sandwich beam theory and an extended theory of the approximate solution by Wölfel. The basics of both theories were explained

and several boundary conditions could be used to simplify the theories. The comparison of the experimental results in the four-point bending test with the calculated results possesses a general application of the calculations for a first assessment of the structural behaviour under static loads. Nevertheless, with the simplification of a linear-elastic material behaviour and a disregard of geometrical non-linear calculation, the analytical method provides conservative results regarding to experimental values. However, it is necessary to detect the real thickness of each layer for an exact analytical calculation. Further research determines the Young's modulus of the thermoplastic polyurethane interlayer with partial composite, so that the calculated results show a sufficient linear modelling of the laminates. The application of the sandwich plate method for a two-sided linearly supported plate according to the four-point bending test does not offer a simple calculation of thin glass-polycarbonate composite panels and complicates the analytical methods. However, the validation of the sandwich beam theory for the four-point bending test with thin glass-polycarbonate composite panels leads to an extension of the two-sided linearly supported composite panel to a four-sided supported plate and the calculation with the plate theory analogously to the calculation methods of laminated glass. The results of a calculation with the plate theory or with finite element software are not content of the presented study.

The benefits of the combination of brittle glass with ductile and impact resistant polycarbonate are located in enhanced requirements of the composite panel. According to EN ISO 12543-4 (2011) and EN 14449 (2005) it is possible to classify the thin glass-polycarbonate composite panel as a laminated safety glass. The composite panel passes the test for durability under high temperature at +100 °C for 16 h, under humidity at 80 % rH and +50 °C for 14 d as well as under radiation in a sun-like spectrum for 2,000 h without any defects, blisters or cloudiness. The confirmation of a laminated safety class also needs the investigation of the behaviour against mechanical loads. The specimens comply all requirements of the ball drop test and an adequate post-breakage behaviour. Further research on a resistance against manual attack according to EN 356 (2001) leads to slighter security glazing in comparison to conventional security glazing made by a glass laminate. The composite panel with the highest resistance class against manual attack P8B achieves 39 % thinner cross-sections and 68 % lower weight. Due to the slim cross-section, the laminate enables thinner frame designs and supporting structures. Retrofitting in existing buildings with high security demands and processing to insulation glass will be easier.

6. Acknowledgements

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