

An Adjustable Mould for the Casting of Glass Voussoirs for the Construction of Fully Transparent Shell Structures

Felix van der Weijst^a, Faidra Oikonomopoulou^a, Marcel Bilow^a

^a*Delft University of Technology, The Netherlands, felixvanderweijst@gmail.com*

Compared to flat sheets of float glass, cast glass components have a thicker geometry and thus a high buckling resistance. This buckling resistance in combination with the high compressive strength of glass make cast glass components suitable for the construction of fully transparent shell structures that are mainly subjected to compressive stresses. Shell structures often have the shape of surfaces with varying Gaussian curvature. When constructing such a shell structure out of cast glass components, components of varying geometries are needed. An adjustable mould was developed that can be used for the casting of glass components (i.e. voussoirs) of varying geometries. The possible voussoir geometries that can be cast in the adjustable mould are limited to voussoirs with planar, convex, polygonal intrados and extrados. These voussoirs can be used to construct fully transparent shell structures. The voussoirs are dry-assembled with a rubber interlayer in between. Tongue and groove shaped interfaces ensure an interlocking connection. By tessellating a shell structure, the shell structure is divided into a discrete number of voussoirs that can be cast in the adjustable mould. Several aspects have to be taken into account when optimizing the tessellation pattern including planarity, interior angle size, face size and alignment to the flow of forces. A design was made of a shell structure that covers the courtyard of the Armamentarium in Delft. This shell structure served as a case study used to demonstrate the tessellating process and the use of the mould developed during this research.

Keywords: Cast glass, Shell structures, Structural glass, Adjustable mould, Voussoir, Tessellation, Dry-assembly

1. Introduction

Since glass is a material with a high compressive strength compared to its tensile strength, its structural potential would be fully utilized when used to construct shell structures that are predominantly loaded in compression. Two domes of structural float glass have been constructed by Veer et al. (2003) and by Blandini and Sobek (2014) of 5.5 and 8.5 meter spans respectively. When constructing shells structures with larger spans, the risk of bucking failure increases. To avoid this risk, either stiffening elements that reduce transparency need to be applied or the thickness of the glass need to be increased to a thickness that could clash with production standards of float glass (Bristogianni et al., 2016). A solution to this problem would be to use cast glass which has a higher thickness compared to float glass and thus a higher buckling resistance.

Since the annealing time of cast glass components exponentially increases as the volume of the components increases, components with a high volume are economically inefficient to produce (Oikonomopoulou et al., 2017). By tessellating a shell structure, the shell can be subdivided into a discrete number of voussoirs that have a limited volume. Since shell structures (with the exception of barrel vaults and spherical domes) have a geometry with varying Gaussian curvature, the tessellation of a shell results in voussoirs with varying geometry. Casting all these different voussoirs in single-use, disposable silica plaster moulds would be a time-consuming production process and would result in voussoirs with a low surface quality (Oikonomopoulou et al., 2018a). An adjustable steel mould is designed in order to facilitate an efficient production process and high surface quality of the voussoirs.

The casted voussoirs can be configured in a shell structure by dry-assembly. Such interlocking dry-assembled cast glass systems have been described in detail by Oikonomopoulou et al. (2018b). These systems have the benefit over adhesively and mechanical bonded systems by being fully transparent and being easy to assemble, disassemble and recycle. A transparent rubber interlayer in between the components prevents peak stresses due to glass to glass contact and compensates the inevitable production tolerances of the glass components.

2. Adjustable mould

Fig.1, Fig. 2, and Fig. 3 show the proposed adjustable mould in triangular, quadrangular and hexagonal configuration. The mould can be used for the casting of glass voussoirs with planar, convex, polygonal intrados and extrados and with interlocking tongue and groove interfaces (the intrados and extrados are the faces of a voussoir that are facing the interior and exterior of the shell. The interfaces are the surfaces of a voussoirs that are in contact with the adjacent voussoirs). The mould consists of different modules that can be categorized into three different types; edges modules, vertex modules and diagonal modules. Multiple edge modules can be joined together by vertex modules. The edge lengths can be adjusted by sliding the vertex modules along the edge modules. The edge modules contain t-slots that allow for the fixing of the vertex modules at a certain edge length with a t-nut and bolt connection. The vertex modules act as hinges. For moulds made up of three edge modules and three vertex modules, the interior angles are automatically adjusted when the edge lengths are changed. For the casting of voussoirs with non-triangular intrados

and extrados, diagonals are added to the mould. This gives indirect control over the interior angles by setting up the length of the diagonals.

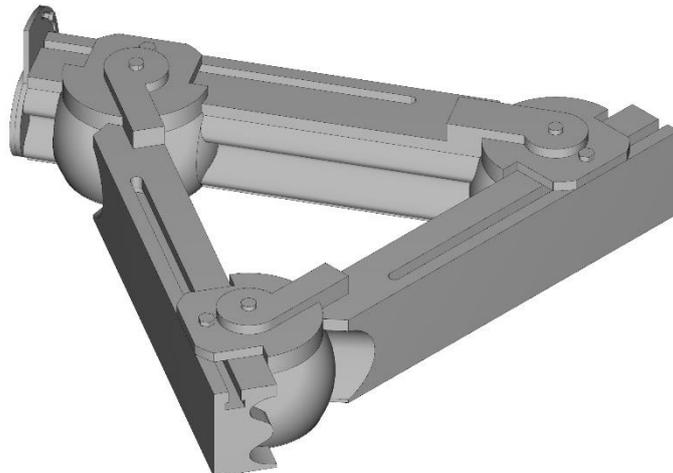


Fig. 1 The adjustable mould in triangular configuration.

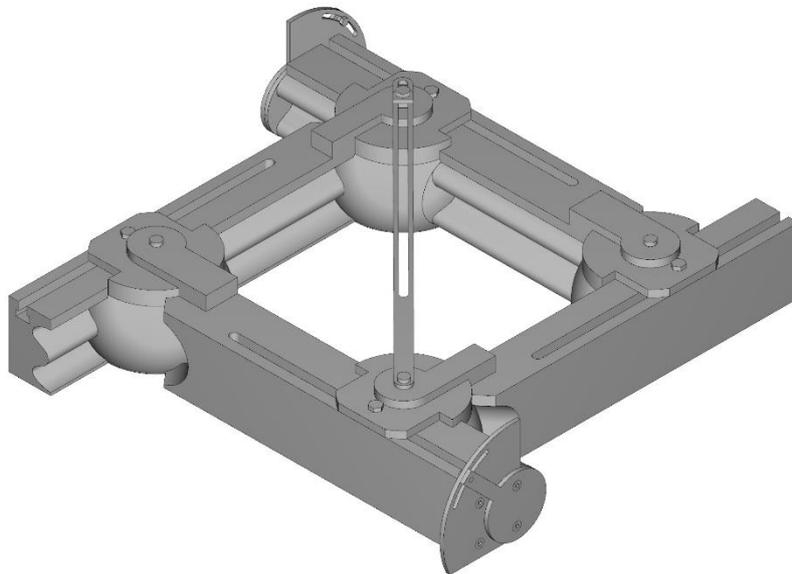


Fig. 2 The adjustable mould in quadrangular configuration.

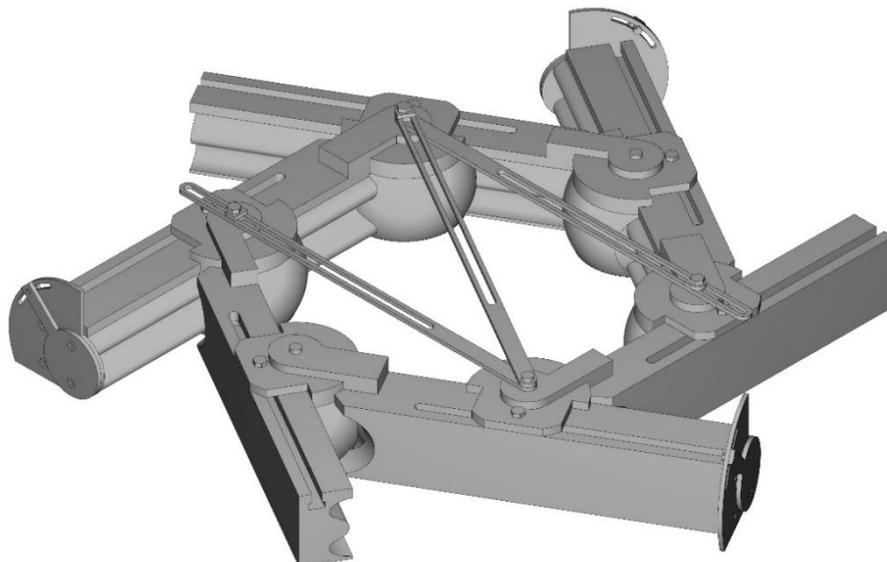


Fig. 3 The adjustable mould in hexagonal configuration.

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Two different edge modules can be distinguished; tongue edge modules and groove edge modules (Fig. 4). These are accompanied by two different vertex modules; tongue vertex modules and groove vertex modules. The different edge modules allow for the casting of voussoirs with interlocking tongue-and-groove interfaces. The tongue edge module consists of two parts; a ‘stationary’ part and a cylinder containing a notch. The notch will give shape to the tongue of the voussoir when casting the glass. The angle between the tongue and the intrados/extrados can be adjusted by rotating the cylinder of the tongue edge module (Fig. 5). This allows for the joining of adjacent voussoirs under varying voussoir joining angles (Fig. 6) which is necessary for the construction of double curved shell structures.

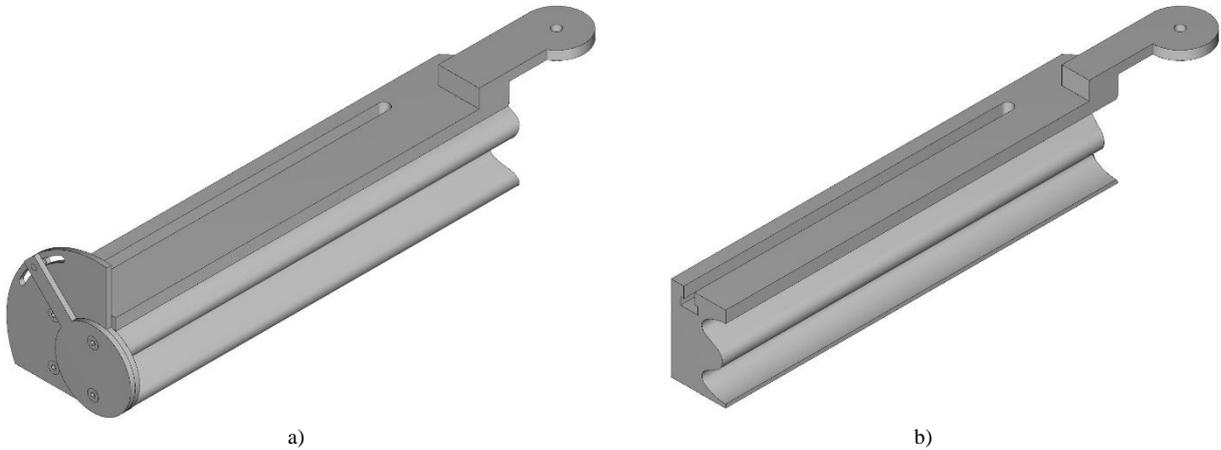


Fig. 4 a) Tongue edge module b) Groove edge module.

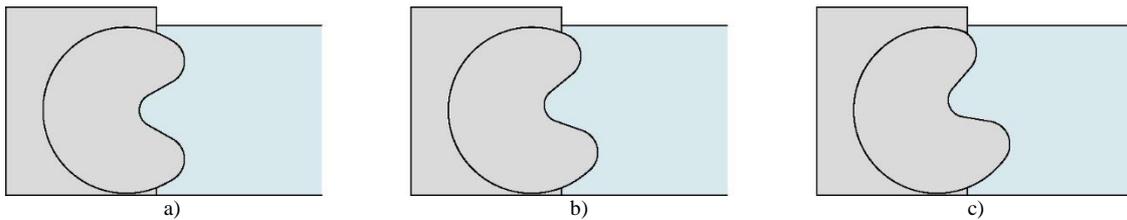


Fig. 5 Tongue edge module set up for tongue interfaces with a voussoir joining angle of a) 0°, b) 10° and c) 20°.

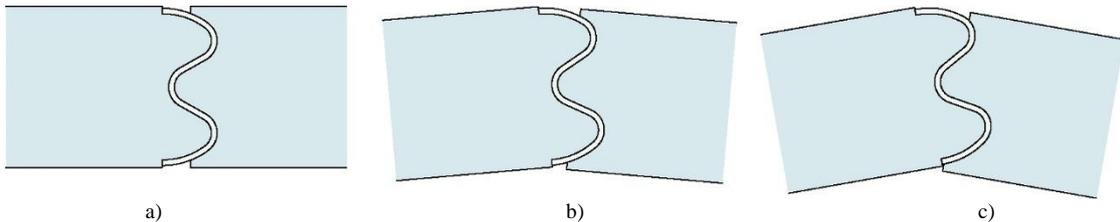


Fig. 6 Dry-assembly of two voussoirs at a voussoir joining angle of a) 0°, b) 10° and c) 20°. Note the interlayer in between the interfaces.

There are two types of vertex modules that can slide along the edge modules; tongue vertex modules and groove vertex modules (Fig. 7). Due to the spherical hinges of the edge modules, the voussoirs will have spherical cavities between the interfaces. A shell structure constructed with these voussoirs will contain spherical holes at the nodes (Fig. 8a). These holes are all identical and can be sealed with spherical node components (Fig. 8b) or spherical segment node components (Fig. 8c). Since the node components all have identical geometry, all node components can be cast in the same mould. The assembly of the voussoirs combined with the node components forms a staggered bond. Each voussoir and node component is supported by at least three other voussoirs or node components and in turn is supporting at least three other voussoirs or node components. A staggered bond allows for the distribution of load inequalities and ‘ties the structure together’ (Rankine, 1889).

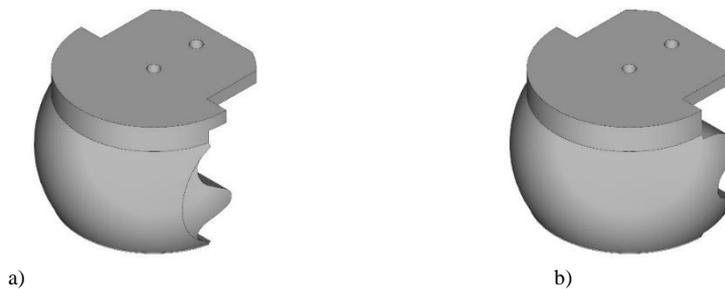
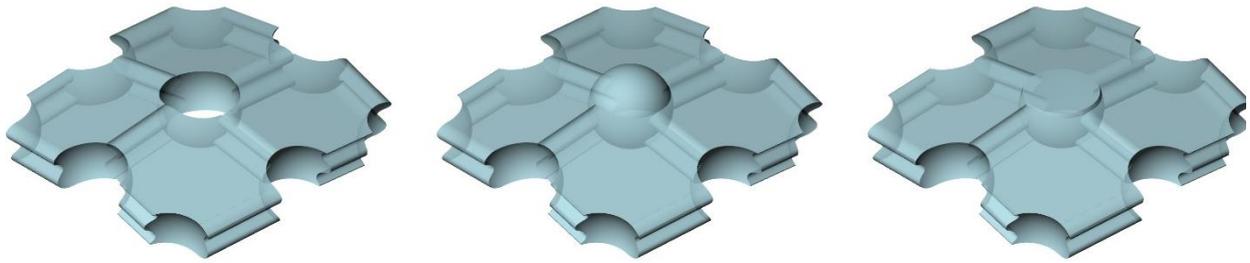


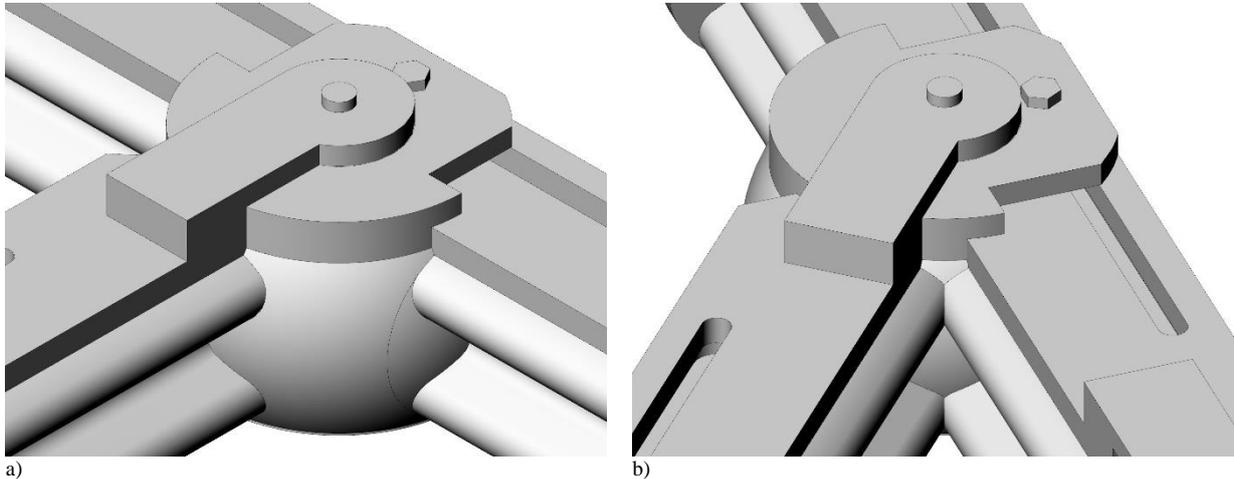
Fig. 7 a) Tongue vertex module b) Groove vertex module.



a) Fig. 8 a) Spherical hole at the node b) Hole sealed with a spherical node component c) Hole filled with spherical segment node component.

2.1. Dimensions

The dimensions of the mould modules are dependent on the geometry of the voussoirs. The height of the mould is equal to the thickness of the voussoirs which is determined by structural requirements of the shell structure. Less obvious is that the radius of the spheres of vertex modules is also influenced by the thickness of the voussoirs. Fig. 9 shows the connection of a tongue edge module to another tongue edge module at a 90° and 40° interior angle. For the 90° connection no problems occur but for the 40° connection the edge modules are intersecting. This problem can be solved by increasing the radius of the sphere of the vertex module. The minimum radius can be calculated with equation 1 wherein r_v is the radius of the sphere of the vertex module, t_v is the voussoir thickness. $\alpha_{i, min}$ is the minimum occurring interior angle of the intrados/extrados of all voussoirs in the entire shell structure.



a) Fig. 9 Tongue edge module connected to a tongue edge module at a) 90° and b) 40°.

$$r_v > \frac{t_v}{2 \sin\left(\frac{1}{2} \alpha_{i, min}\right)} \quad (1)$$

Note that this problem of edge module intersection at small angles only occurs for tongue edge module to tongue edge module connection. Instead of solving this problem by increasing the sphere radius of the vertex module, it can also be solved by making sure that for small angles only tongue to groove edge module, groove to groove edge module or tongue to groove edge module connections are needed.

A similar problem occurs for large angles. Fig. 10 shows the connection of groove edge module to a tongue edge module at 90° and 140° interior angle. For the 90° connection no problems occur but for the 140° connection the edge modules are intersecting (left of the vertex module in Fig. 10b. This intersection also occurs when connecting a tongue edge module to another tongue edge module at a 135 angle. This problem can be solved by increasing the radius of the sphere of the vertex module. The minimum radius can be calculated with equation 2 wherein r_v is the radius of the sphere of the vertex module, t_v is the voussoir thickness and $\alpha_{i, max}$ is the maximum occurring interior angle of the intrados/extrados of all voussoirs in the entire shell structure.

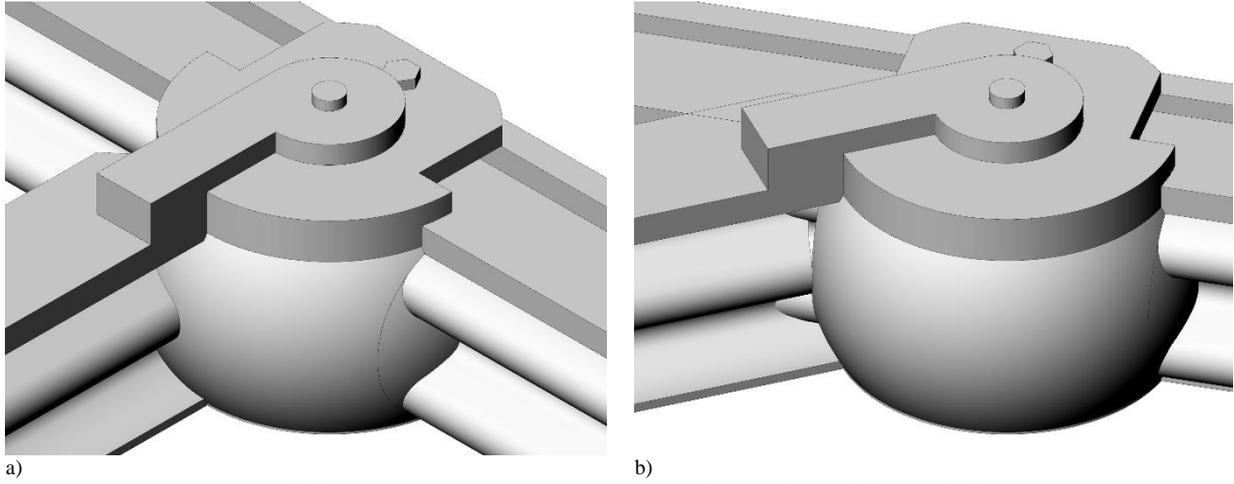


Fig. 10 Groove edge module connected to a tongue edge module at a) 90° and b) 140°.

$$r_v > \frac{t_v}{2 \cos\left(\frac{1}{2} \alpha_{i,max}\right)} \quad (2)$$

Note that this problem of edge module intersection at large angles only occurs for groove edge module to tongue edge module connections. As with the problem for small angles, the problem can be solved by increasing the sphere radius of the vertex module but also by avoiding certain connections at large angles. For large angles, tongue to tongue edge module, groove to groove edge module or tongue to groove edge module connections are preferred.

3. Tessellation

As mentioned before, the shell structure needs to be subdivided into multiple castable components of limited size to limit the annealing time of each component. By tessellating the shell structure, the shell is divided in a discrete number of faces. Each face represents a voussoir. Three regular tessellation patterns can be distinguished: triangular, quadrangular and hexagonal. A comparison of the tessellation pattern is made based on constraints related to the adjustable mould and structural performance.

3.1. Planarity and convexity

The use of the adjustable mould is limited to the casting of voussoirs with planar, convex polygonal intrados and extrados. A benefit of triangular tessellations patterns is that the faces are inherently planar and convex. Quadrangular and hexagonal tessellations are not guaranteed have faces that are all planar and convex but algorithms have been developed to convert these tessellations into tessellations with planar, convex faces. However, hexagonally tessellated surfaces that contain regions of negative Gaussian curvature (e.g. hyperboloid shells structures), can only be planarized with the incorporation of concave faces (Wang & Liu, 2009).

3.2. Interior angles

Another mould related factor that has to be taken into account when tessellating a shell structure are the interior angles of the faces and the length of the edges. As discussed in §2.1, the radius of the sphere of the hinge module is determined by the thickness of the voussoirs and the interior angles of the intrados/extrados. The decisive interior angle is either the smallest interior angle that occurs in the tessellation pattern at which a tongue edge module is connected to another tongue edge module, or by the largest interior angle that occurs in the tessellation pattern at which a groove edge module is connected to a tongue edge module. Whichever of these angles has the largest deviation from 90° determines the radius of the sphere of the vertex module and node components. In order to avoid the intersecting of two node components, all edges of the tessellation need to have a length larger than twice the radius of the node components. The faces of quadrangular tessellations have an average interior angle of 90°. When the tessellation is properly generated, the minimum and maximum angle will be close to 90°. However, the minimum and maximum interior angles of the faces of triangular and hexagonal tessellation will be smaller than 30° or larger 60° respectively. Therefore, the node components will be larger in triangular and hexagonal tessellations than in quadrangular tessellations. As a result, complications due to edges being smaller than twice the radius of the node components are more likely to occur in triangular and hexagonal tessellations than in quadrangular tessellations.

3.3. Alignment to the flow of forces

The final tessellation constraint is related to the structural performance of the tessellated shell structure. Ideally, tessellation patterns are orientated in such a way that the interfaces are as perpendicular to the flow of forces as possible (Rankine, 1889). When the angle between the flow of forces and the interfaces decreases, the amount of friction between the voussoirs decreases and sliding failure might occur. The orientation of the tessellation pattern is less relevant for shell structures that are fully supported along their boundaries (e.g. conventional domes). Each voussoir is locked in place by the adjacent voussoirs and by the supports. However, for shell structures with unsupported

openings (e.g. barrel vaults), a non-properly orientated tessellation pattern can result in voussoirs sliding towards the openings of the shell causing a collapse of the shell (Rippmann & Block, 2018).

Due to the geometrical nature of a quadrangle, all edges of a quadrangular face have the potential to be aligned with one of the two principal force directions. This is not the case for triangular or hexagonal faces where the perfect alignment to the principal force direction of two edges, automatically results in the remaining edges being not aligned. Fig. 11 shows three barrel vaults with three different tessellation patterns. The quadrangularly tessellated vault has a tessellation pattern that is perfectly aligned with the flow of forces and no risk of sliding occurs. For the triangularly and hexagonally tessellated vaults, the tessellation pattern is not properly orientated with the flow of forces resulting in the voussoirs at the unsupported boundaries being at risk of sliding.

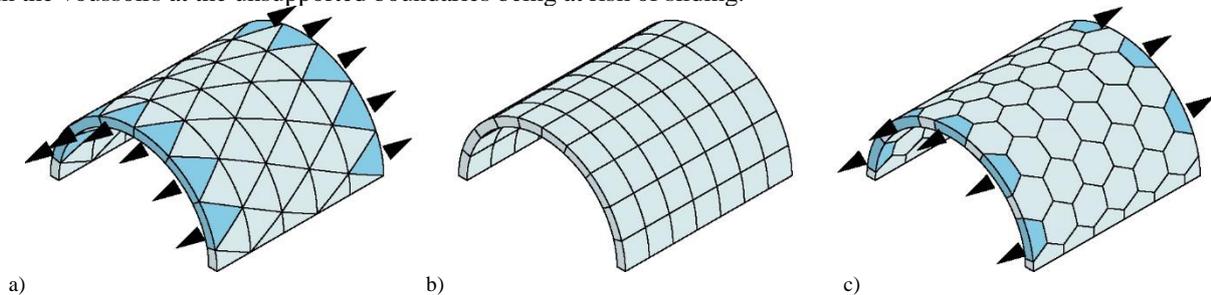


Fig. 11 Barrel vault with a) a triangular tessellation pattern with a risk of sliding, b) a quadrangular tessellation pattern and c) a hexagonal tessellation pattern with a risk of sliding.

The triangular and hexagonal tessellations patterns can be redesigned in such a way that they are better orientated to the flow of forces (although not perfectly orientated), and thus preventing the voussoirs at the unsupported boundaries from sliding (Fig. 12). Note that the redesigned tessellation patterns have faces with altered interior angles resulting in larger node components. As described in §2.1, these larger node components lead to complications when their radius is larger than half the minimum edge length.

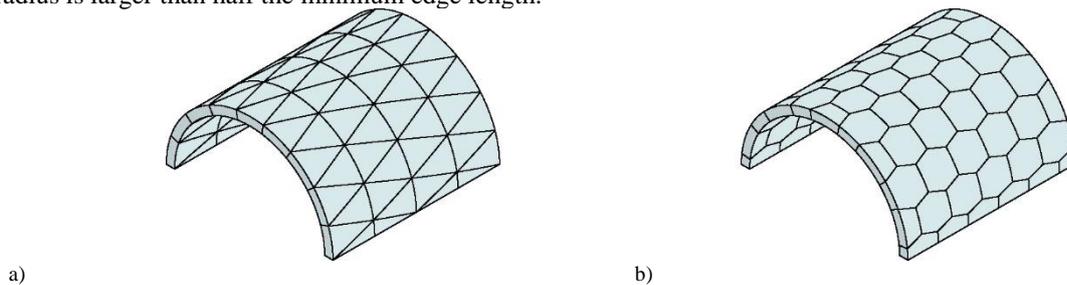


Fig. 12 Barrel vault with a) a redesigned triangular tessellation pattern without risk of sliding and b) a redesigned hexagonal tessellation pattern without risk of sliding.

4. Case study

In order to demonstrate the use of the adjustable mould, a shell structure was designed that acts as a covering for the courtyard of the Armamentarium in Delft (Fig. 13). The dimensions of the courtyard are approximately 10.3 by 16.5 meters. The shape of the shell was generated using the particle-spring method. In order to determine the required thickness of the voussoirs, the shell structure has to be subjected to a structural analysis. Since the shell structure consists of dry-assembled voussoirs (i.e. discrete shell structure) instead of being monolithic, a conventional FEM analysis alone is not sufficient to validate the structural performance. FEM analyses are suitable to analyse stresses and displacements but not for analysing the stability of discrete shell structures. For shell structures on the other hand, the structural integrity is mainly dependent on the stability while stresses are of lesser concern since these are usually limited to a small percentage of the yield strength (Heyman, 1995). Research by Van Mele et al. (2012) has shown that discrete element modelling (DEM) is a suitable method for analysing the stability of discrete shell structures. DEM is a computational method capable of analysing the interaction between separate bodies while taking the friction between the bodies in account. Because the author is not familiar with DEM and the rubber interlayer in between the voussoirs further complicates the analysis, and because the structural analysis of discrete shell structures is not the focus of this research, the thickness of the voussoirs is assumed to be 100 mm. This turns out to be a very conservative assumption when comparing the 100 mm thickness and 10.5 meter span of the case study with the Armadillo Vault by Van Mele et al. (2016) that has a varying thickness of 50 to 120 mm and a 15 meter span.

A triangular tessellation pattern was selected for subdividing in shell in multiple voussoirs. Since the shell is fully supported along its boundary, no risk of sliding occurs when the tessellation pattern is not properly orientated with the flow of forces and distorted triangular faces with very sharp interior angles are avoided. Furthermore, by selecting a triangular tessellation pattern, the complex process of planarization could be circumvented. The constraints for the tessellation were as following:

- Maximize the smallest occurring edge length to avoid the intersection of node components
- Maximize the smallest occurring interior angle to reduce the radius of the node components
- Minimize the area of each face to reduce the mass of the voussoirs

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These are conflicting constraints since an increase in edge lengths and interior angles will lead to increased face areas and thus to voussoirs with a higher mass. Theoretically, the radius of the spheres can be reduced by decreasing the thickness of the voussoirs, but this will conflict with the structural requirements. Since non-intersecting node components and sufficient voussoir thickness are essential constraints to make this structural system function properly, these constraints are prioritized over the minimalization of the mass of the voussoirs. This will come at the cost of an increased annealing time of the voussoirs.

The tessellation pattern was generated using T.MAP, a mesh parametrization tool developed by EvoluteTools. The tessellation was further post-processed by hand to increase the minimum occurring edge length and minimum occurring interior angles. The resulting tessellation pattern had 1250 faces, a minimum occurring edge length of 330 mm and a minimum occurring interior angle of 34.2°.

An algorithm was developed to assign the type of interface (tongue or groove) to each edge of all faces. The algorithm contained a constraint to avoid the assignment of two tongue interfaces to the face edges joining at the smallest occurring interior angles wherever possible. After the algorithm assigned the type of interfaces to each edge of all faces, the smallest angle at which two face edges to which both tongue interfaces were assigned turned out to be 44.8°. With equation 1, the minimum radius of the vertex edge module, r_v , was calculated to be 132 mm. Since r_v is smaller than twice the minimum occurring edge length (330 mm), no complications due to intersecting node components occur.

Based on the tessellation pattern and the assignment of the interface types, the input data for the adjustable mould was generated. The input data consists of the following information:

- Voussoir index number
- For each edge:
 - Type of interface (tongue or groove)
 - Edge length
 - Voussoir angle (only for edges with a tongue interface)
 - Adjacent voussoir index number

The mass of each voussoir was calculated. The average mass of the voussoir is 24.6 kg and the voussoirs of lowest and highest mass weighing 5.41 kg and 38.5 kg respectively. Comparing these voussoirs to the soda-lime glass bricks of the Crystal house (7.2 kg, 65 mm × 210 mm × 210 mm) that required 36-38 hours to anneal (Oikonomopoulou et al., 2017) and the borosilicate glass bricks of the Atocha Memorial that required (8.4 kg, 70 mm × 200 mm × 300 mm) that required 20 hours to anneal (Paech & Goppert, 2008), it is estimated that the annealing time of the average voussoir is more than 50 hours, even if borosilicate glass is used.

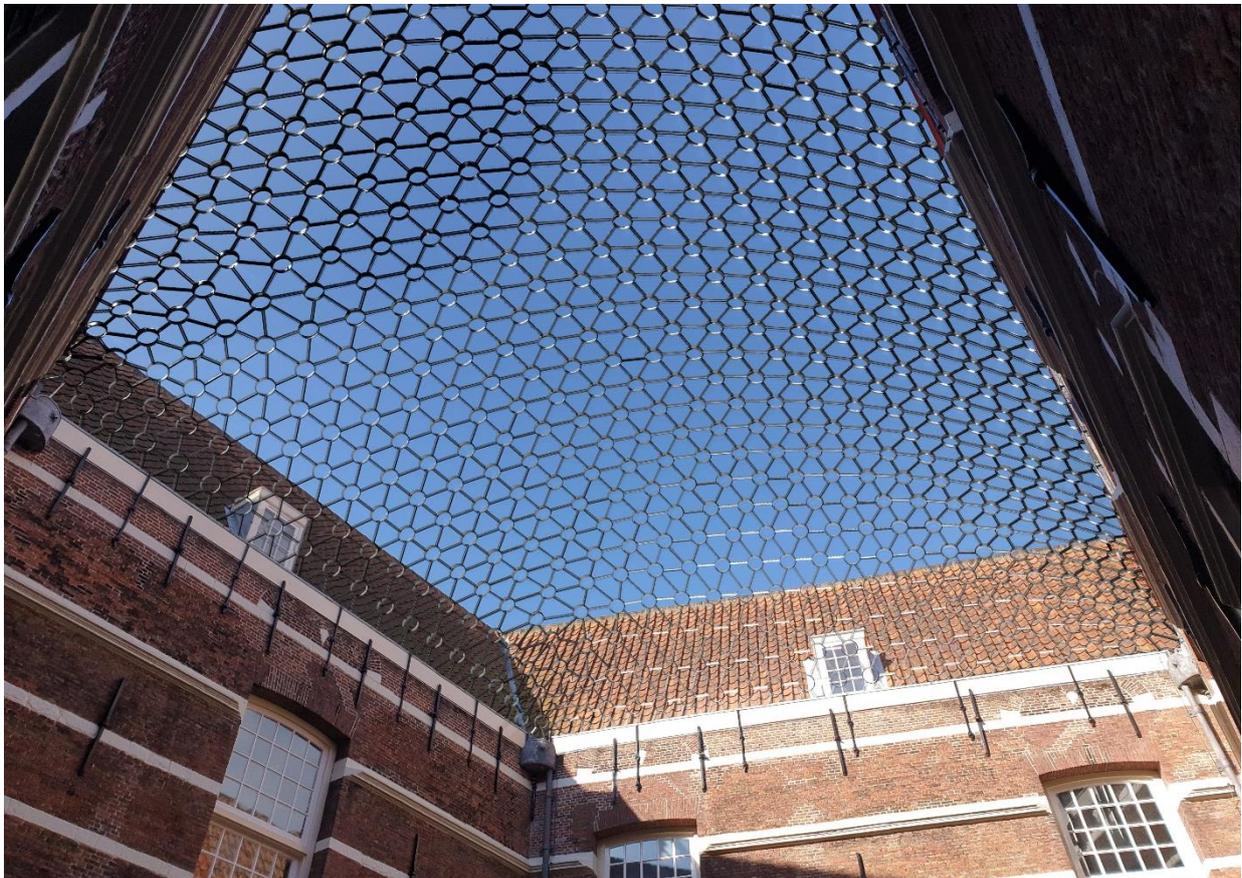


Fig. 13 Design for a cast glass shell structure covering the courtyard of the Armamentarium in Delft.

5. Conclusions

This research has resulted in a production process and a set of design constraints for cast glass shell structures. An adjustable mould has been developed that is suitable for the casting of glass voussoirs with planar, convex intrados and extrados. The voussoirs contain interlocking tongue and groove interfaces allowing for a dry-assembled bond between the voussoirs. A shell made of these voussoirs contains spherical holes that can be sealed with spherical node components. Furthermore, the combined assembly of voussoirs and node components forms a staggered bond.

A set of design constraints needs to be taken into account when tessellating the shell structure into multiple voussoirs. Due to the limitations of the mould, all faces in the tessellation need to be planar and convex. Faces with very small or very large interior angles should be avoided and all edges need to be at least twice as long as the radius of the node components. The face area of the voussoirs should be limited to avoid voussoirs with a high mass that require a long time to anneal. Finally, when the shell contains unsupported edges, the tessellation pattern should be aligned to the flow of forces to avoid collapse due to sliding of the voussoirs at the unsupported edges.

6. Limitations and recommendations

The structural system of interlocking voussoirs combined with spherical node components is based on the research on interlocking cast glass structures with a rubber interlayer in between by Oikonomopoulou et al. (2018b). Such structures are currently only in the domain of academic research and have not yet been applied in the built environment. Certain features of this structural system require further research, such as the rubber interlayer (Oikonomopoulou et al., 2019). Further research should confirm the feasibility of interlocking cast glass structures before further research on the adjustable mould and cast glass shell structures is done.

This research is limited to the development of a conceptual production process for cast glass voussoirs. To validate this concept, a prototype of the mould should be build and used for casting voussoirs. The casting of the voussoirs can give insight in the annealing time and the residual stresses in the voussoir. Furthermore, the casted voussoirs can be subjected to compression and bending tests to assess the structural performance of the individual voussoirs and the dry-assembled tongue and groove connection.

The conservative estimation of the thickness of the shell designed as a case study, resulted in voussoirs with a high mass and long annealing time. This will presumably render the structure economically infeasible. If the proper method for structural analysis (DEM) is used, it is likely that a lower thickness will prove to be sufficient for the structural performance and thus results in voussoirs with a lower mass and annealing time. Another way to reduce the mass is by further optimizing the tessellation pattern to consist of smaller faces. However, this should be done with respect to the stated tessellation constraints. It could be worth to develop an algorithm that generates the tessellation pattern while carefully balancing the sometimes conflicting constraints.

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