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Failure Criteria for SentryGlas® Ionomer and TSSA Silicon: A Theoretical Introduction to a Novel Generalized Triaxial Model (GTM)

Manuel Santarsiero [a] & Christian Louter [b]
[a] *Eckersley O'Callaghan Engineers, United Kingdom, manuel.santarsiero@gmail.com*
[b] *Delft University of Technology (TU Delft), Faculty of Architecture and the Built Environment (A+BE), Department of Architectural Engineering and Technology (AE+T), Chair of Structural Design, The Netherlands*

SentryGlas® (SG) (a transparent ionomer from Kuraray) and Transparent Structural Silicon Adhesive (TSSA) (a transparent silicon from Dow Corning) are two of the adhesive materials used in laminated adhesive connections for structural glass applications. Although they have been used in several projects worldwide, failure criteria for these materials are currently not available in literature. This work gives an introduction to the theoretical development of a novel failure a criterion for TSSA and SG under varying stress state conditions. The main output of this work is a four-dimensional Generalized Triaxial Model (GTM) that accounts for a generic stress state by a governing equation expressed as a function of the three-dimensional stress tensor. Both deviatoric and hydrostatic energetic components are taken into consideration by means of a non-linear function of the two contributions. The effects of the model parameters are investigated by a parametrical study. The proposed model is then analytically compared to existing failure criteria. It is shown that many of the existing models available in the literature can be seen as particular cases of the proposed model. Although the GTM was theoretically developed specifically for SG and TSSA, the model can be generically used as failure criterion for many isotropic materials.

Keywords: SentryGlas®, ionomer, TSSA, silicon, laminated connections, adhesive, failure criteria, analytical model

1. Introduction

The SentryGlas® (SG) (a transparent ionomer from Kuraray) and the Transparent Structural Silicon Adhesive (TSSA) (a transparent silicon from Dow Corning) are two of the transparent adhesive materials used in laminated adhesive connections for structural glass applications. Although they have been used in several projects worldwide (Sitte et al. 2011; O'Callaghan 2012; Lenk & Lancaster 2013; Peters et al. 2007; Puller et al. 2011; Santarsiero et al. 2013; Carvalho et al. 2011; Ludwig 2015), failure criteria for these materials are currently not available in literature. This work gives an introduction to the theoretical development of a novel multi-dimensional model that accounts for any generic stress state. This is done by developing a Generalized Triaxial Model (GTM) defined over the three-dimensional stress space and that accounts for the non-linear effects of strain rate and temperature variation. The theoretical bases and main equations that are necessary for a concise analytical description of failure criteria are given. Stress state equations in the Haigh-Westergaard space are derived in orthogonal and cylindrical coordinates. The GTM is developed as a function of the three-dimensional stress space, strain rate and temperature. A parametrical study is then performed to investigate the effects of the model parameters. The proposed model is then compared to existing models and the main theoretical hypotheses are highlighted.

2. Theoretical framework

The analytical bases necessary for the definition and the mechanical interpretation of failure criteria are given in this section. In particular, three different stress-space coordinate systems are derived. Basic concepts of solid mechanics are assumed to be as known for the sake of brevity.

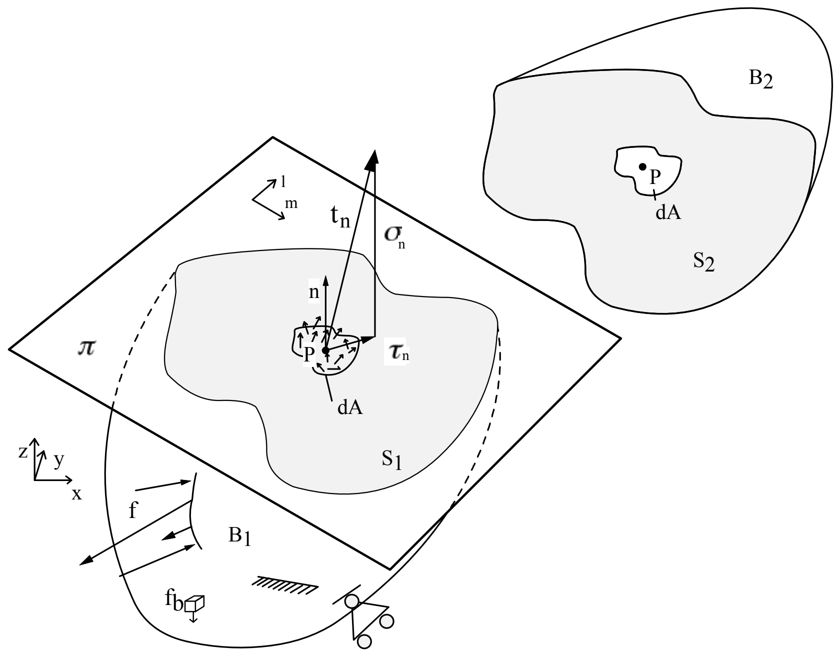


Fig. 1 Scheme of the total stress vector at point P of a solid body under generic boundary conditions, body forces, and surface forces

Let us consider a continuous solid body B, in equilibrium, subjected to boundary conditions Ω, body forces f_b, and surface forces f (Fig. 1). The body is then divided in two parts, B_1 and B_2, by a cutting plane π, containing point P and with normal n. Surfaces S_1 and S_2 are so created. In a x-y-z reference system, the plane π is fully determined by the point P and the normal n, with the latter not being necessarily parallel to one of the axes. Now, to guarantee the equilibrium of each body part, force fields are acting onto the surfaces S_1 and S_2. Before the cutting, these forces were applied by B_1 to B_2 and vice-versa. Considering an infinitesimal part of the surface S, of area dA, centred on the point P, the total stress[1] vector t_n is defined by equation (1) (Gere & Timoshenko 1997). In (1), dR is the vector sum of the forces acting on the surface dA. The same procedure can be repeated for any generic plane π_i containing the point P. Having the full knowledge of the stress state at point P means to know magnitude and direction of the vector t_n for any plane π_i containing the point P.

$$t_n = \lim_{dA \to 0} \frac{dR}{dA} \tag{1}$$

The stress state at point P is fully described by knowing a three-dimensional stress tensor describing three total stress vector derived from an orthogonal reference system. There is at least one orthogonal reference system with zero shear forces along the planes with normals n_1, n_2 and n_3. Consequently, the stress state is fully described by three scalar quantities, σ_1, σ_2 and σ_3, i.e. principal stresses at point P (equation (2)). Alternatively, the stress state is also fully determined if the stress invariants are given (equations (3) or (4)) (Timoshenko 1930a).

$$\sigma_{ij} = F(\sigma_1, \sigma_2, \sigma_3) \tag{2}$$

[1] It is considered a solid body that also satisfy the condition of having the momentum of R with respect to P equal to zero, i.e. B is non-polar solid.

$$\sigma_{ij} = F(I_1, I_2, I_3) \tag{3}$$

$$\sigma_{ij} = F(I_1, J_2, J_3) \tag{4}$$

Now, let us consider a three-dimensional space, known as Haigh-Westergaard stress space, with a principal reference system n_1, n_2 and n_3 centred at point P (see Fig. 2). The stress state is described by the position of the point P_s, which is expressed in orthogonal coordinates σ_1, σ_2 and σ_3. Notice that the vector PP_s does not necessarily represent the total stress vector at P but rather the graphical representation of the three-dimensional stress state at P in the stress H-W space. With the purpose of describing the derivation and main characteristics of failure criteria, it is now useful to express the stress state PP_s in cylindrical coordinates. To do so, the hydrostatic axis and the deviatoric planes are defined using the definitions of deviatoric and hydrostatic components of the stress tensor. Namely, the hydrostatic axis, ξ, is defined as the direction of the vector sum of n_1, n_2 and n_3. The deviatoric plane is instead defined as the plane orthogonal to ξ (see Fig. 2 (a)).

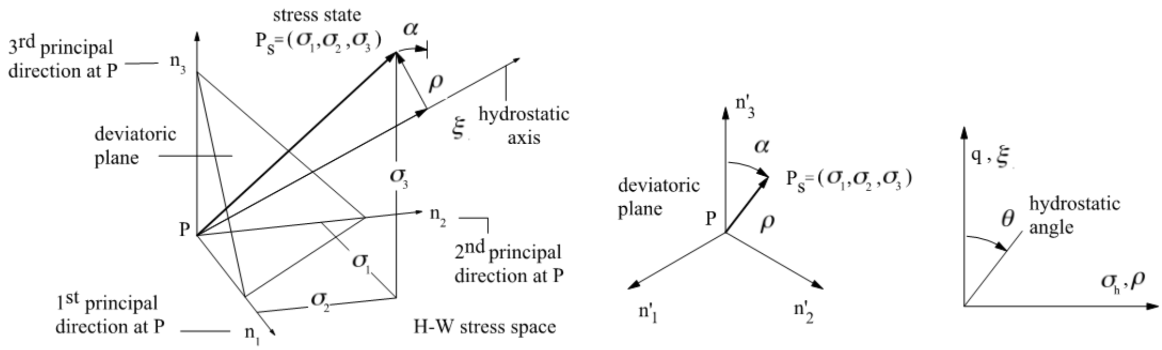


Fig. 2 Representation of stress state at P in the three-dimensional Haigh-Westergaard stress space, in an orthogonal (*x,y,z*) and a cylindrical (ξ, ρ, α) reference system

$$\alpha = \frac{1}{3}\, arcos\left(\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right) \tag{5}$$

Now, using the cylindrical coordinate system, the stress state is know when are given the coordinate along the hydrostatic axis ξ, the distance from the hydrostatic axis ρ, and the angle of its projection α, over the deviatoric plane with any of the principal axis projection over the deviatoric plane (see Fig. 2 (b)). In Fig. 2 (b), n_1', n_2' and n_3' represent the projections of the principal axes over the deviatoric plane. The definition of α, also called Lode's angle (Lode 1926), is given by equation (5). It should be noticed that the coordinate ξ is not equal to the hydrostatic stress (σ_h). In that respect, it is then convenient to define a second cylindrical reference system. Same definitions of hydrostatic axis and deviatoric plane apply, while the hydrostatic stress σ_h is now used instead of ξ and the equivalent stress (here indicated as q) is used instead of ρ. The three-dimensional stress state at point P can be now expressed in the form of equations (6) and (7).

$$\bar{\sigma} = F(\xi, \rho, \alpha) \tag{6}$$

$$\bar{\sigma} = F(\sigma_h, q, \alpha) \tag{7}$$

Considering now the three-dimensional stress space of Fig. 2, a material failure criterion is generally defined as the analytical envelope of all admissible stress state of the body B at point P. A failure criterion therefore represents the three-dimensional analytical surface[2] containing all possible configurations of vector PP_s. Making use of the previous considerations on the stress state tensor, a failure criterion is therefore generally expressed by equation (8). In (8), g represents the governing equation of the failure criterion, which is a function of the stress tensor elements or, equivalently, of the principal stresses.

[2] Analytical constrains usually apply to the governing equation of failure criteria (see following section).

$$g(\sigma_{ij}) = 0$$
$$g(\sigma_1, \sigma_2, \sigma_3) = 0 \tag{8}$$

Accounting for the stress tensor properties and the orthogonal-cylindrical transformation described in this section, any failure criterion can be analytically rearranged in terms of stress invariants or in terms of cylindrical coordinates (equation (9)). This will allow, in the following section, to define the governing equation of failure criteria in a concise form.

$$g(I_1, J_2, J_3\,) = 0$$
$$g(\sigma_h, q, \alpha) = 0 \tag{9}$$

In case of isotropic material, failure criteria are symmetric with respect to n_1', n_2' and n_3'(Timoshenko 1930b). A three-dimensional failure criterion is therefore fully defined when the equation of the surface meridian[3] is given over a Lode's angle span of 60°. In some cases, failure criteria are also independent from the angle α, i.e. characterized by axial-symmetry with respect to the hydrostatic axis. In these cases, only one meridian equation is sufficient to fully define the failure criterion. It follows that, for Lode-independent failure criteria (i.e. J_3-independent), the use of a cylindrical coordinate system allows to synthetize the three-dimensional criterion definition by means of a two-dimensional equation (see Fig. 2 and equation (9)), unless other dimensions are involved (e.g. temperature and strain rate).

3. Generalized triaxial model

3.1. Basic concept

The governing equation of model should be defined as a function both the distortion energy and the hydrostatic energy density. More specifically, based on the experimental observation of (Santarsiero 2015), the model aims to account for a generic non-linear combination of the two deviatoric and hydrostatic contributions. Secondly, the model should not be limited to a specific stress state condition but instead apply to generic configuration of the three-dimensional stress tensor. It follows that it should not apply only to a specific value of triaxiality (ration between hydrostatic and equivalent stress) but instead apply to a varying value of the hydrostatic angle. Thirdly, the model aims to quantify the effects of strain rate and temperature variation. More specifically, based on the results of (Santarsiero 2015), non-linear analytical expressions should be implemented as a function of the strain rate and temperature values.

$$g = F(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{zy}, \tau_{xz}, \tau_{xy}, T, \dot{\varepsilon}, p) \tag{10}$$

$$g = F(\sigma_1, \sigma_2, \sigma_3, T, \dot{\varepsilon}, p) \leq 0 \tag{11}$$

In order to fulfill these conditions, the model assumes the form of equation (10), or equivalently, equation (11). It follows that a 5-dimensional model should be defined, as a non-linear function of the principal stress, temperature, strain rate and p. In equation (11), p is a vector containing a set of material parameters. In this work, as done in von Mises and Drucker-Prager models, an axial-symmetric model is developed, i.e. independent of the third stress invariant. The dimensions of the model therefore reduces from 5 to 4. Subsequently, the general expression of the model assumes the form of equation (12) if expressed in term of stress invariants, or (13) if expressed in term of q and σ_h.

$$g = F(I_1, J_2, T, \dot{\varepsilon}, p) \leq 0 \tag{12}$$

$$g = F(\sigma_h, q, T, \dot{\varepsilon}, p) \leq 0 \tag{13}$$

[3] Intersection of the failure criteria with a generic plane containing the hydrostatic axis

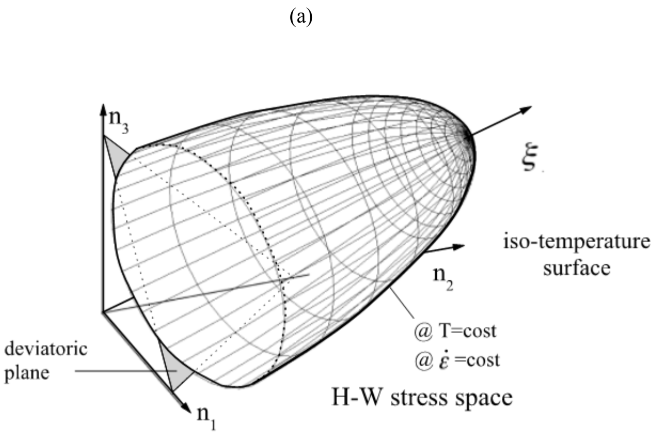
In the following section the analytical definition of the model is given in term of q and σ_h because this allows to provide a mechanical interpretation to the material parameters. Nevertheless, the model can be reformulated in terms of stress invariants or principal stresses

3.2. Analytical definition of the model and parameters effects

The governing equation of the generalized triaxial model (GTM) proposed in this work is given by equation (14). In equation (14), q is the equivalent stress, σ_h is the hydrostatic stress, β_q and β_h are material parameters, and a and b are non-linear functions of the temperature, strain-rate and the material under consideration. The GTM is therefore analytically defined as a function of the stress tensor element, the strain rate and the temperature. Equation (14) also shows that the model is not restricted to a specific value of triaxiality but instead it applies to a varying value of hydrostatic angle, therefore to any generic three-dimensional stress state (i.e. any load condition and geometry).

$$g = q^{\beta_q} + a\,\sigma_h^{\beta_h} - b \leq 0 \qquad \text{with } a(T,\dot{\varepsilon}) \text{ and } b(T,\dot{\varepsilon}) \tag{14}$$

In a three-dimensional stress space, the proposed model represents a revolution surface characterized by an axial symmetry[4] with respect to the hydrostatic axis (Fig. 3 (a)). The revolution surfaces closes at increasing values of hydrostatic stresses with a vertex at coordinate $\xi=(3^{0.5}b/a)^\wedge(1/\beta_h)$). The intersection of the GTM with a plane of zero principal stress (e.g. $\sigma_2=0$) gives an ellipse. The intersection with a deviatoric plan, at a given temperature and strain rate, is a circle with variable radius, function of the hydrostatic coordinate. At increasing temperature and decreasing strain rate, the radius of the iso-hydrostatic circles decreases (see Fig. 3 (b)). In Fig. 3 the model is plotted in the three dimensional stress space, at constant strain rate and temperature.



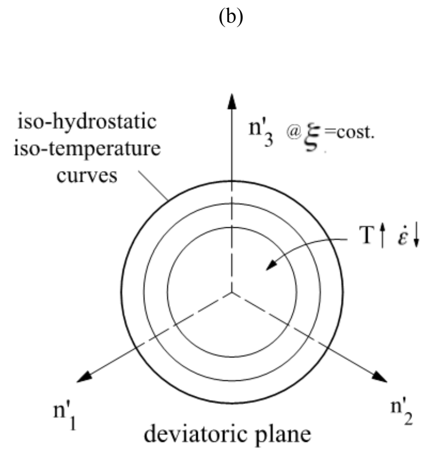


Fig. 3 (a) Proposed generalized triaxial model over the three-dimensional Haigh-Westergaard stress space
(b) Iso-hydrostatic sections on the deviatoric plane at varying temperature and strain rate values

Equation (14) shows that the proposed model is defined as a non-linear function of the two components of the stress tensor. The deviatoric and hydrostatic components are implemented respectively by means of q and σ_h. More specifically, according to equation (14), the deviatoric energy decreases when the hydrostatic stress increases and the hydrostatic energy decreases when the equivalent stress increases. The non-linear interaction of the two stress components is regulated by the parameters β_q and β_h.

[4]Because of this symmetry, only two of the three stress dimensions are independent. Although it applies to a generic three-dimensional stress tensor, the total number of dimensions of the proposed model goes from five to four.

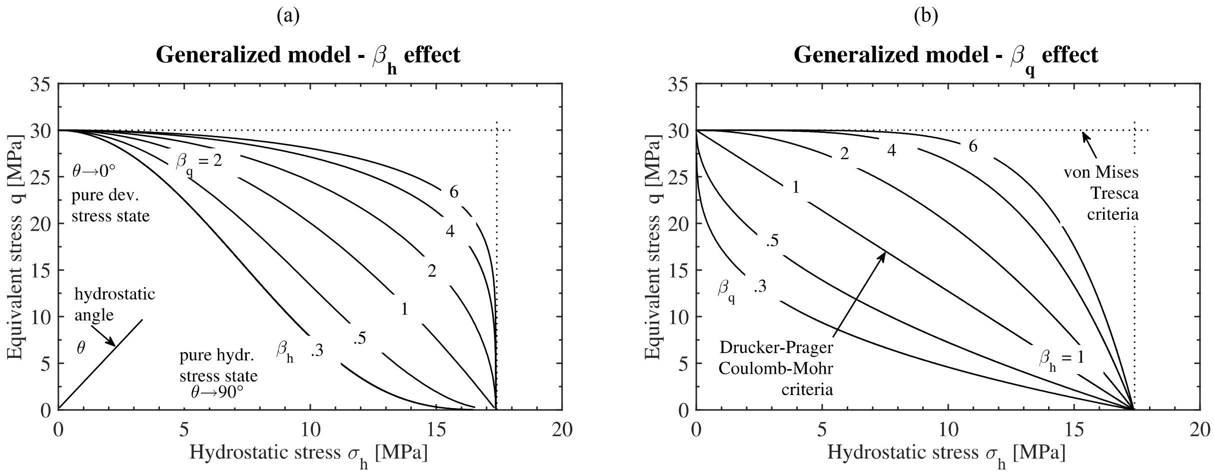


Fig. 4 Effects of β_h (a) and β_q (b) parameters on the meridian curve of the proposed model

The shape of the curve is directly affected by the values of these two parameters β_q and β_h. More specifically, (i) β_h affects the tangent of the curve at deviatoric stress equal to zero, which at increasing β_h rotates clockwise up to vertical position (indicated by vertical dotted line) and (ii) β_q affects the tangent of the curve at hydrostatic stress equal to zero, which at increasing β_q rotates counter clockwise up to horizontal position (indicated by horizontal dotted line). It is also observed that existing models are particular cases of the proposed model (indicated with arrows in Fig. 4 (b)), as discussed more in detail in the following section.

$$\left(\frac{q}{q_0(T,\dot{\gamma})}\right)^{\beta_q} + \left(\frac{\sigma_h}{\sigma_{h,0}(T,\dot{\varepsilon})}\right)^{\beta_h} - 1 \leq 0 \tag{15}$$

$with$:

$$q_0(T,\dot{\gamma}) = q_v \cdot \alpha_{T,v} \cdot \alpha_{\gamma,v} \tag{16}$$

$$\sigma_{h,0}(T,\dot{\varepsilon}) = \sigma_{h,n} \cdot \alpha_{T,n} \cdot \alpha_{\varepsilon,n} \tag{17}$$

The GTM is now analytically rearranged from equation (14) to equation (15). The governing equation can be seen as the sum of two terms: the deviatoric term (first term) and the hydrostatic term (second term). In the derivation of equation (15), the functions a and b are converted in the function q_0 and $\sigma_{h.0}$. The mathematical relationships between these functions are given by equation (18).

$$b = q_0^{\beta_q} \quad ; \quad a = \frac{q_0^{\beta_q}}{\sigma_{h,0}^{\beta_h}} = \frac{b}{\sigma_{h,0}^{\beta_h}} \tag{18}$$

If compared to equation (14), the governing equation of the model rearranged in equation (15) allows to provide a mechanical interpretation to the material parameters. More specifically (i) the parameter q_0 is related to the maximum value of distortion energy density at an hydrostatic angle equal to 90°, i.e. stress state condition with hydrostatic component of the stress tensor equal to zero and (ii) the parameter $\sigma_{h.0}$ is related to the maximum value of hydrostatic energy density at hydrostatic angle equal to 0, i.e. stress state condition with deviatoric component of the stress tensor equal to 0.

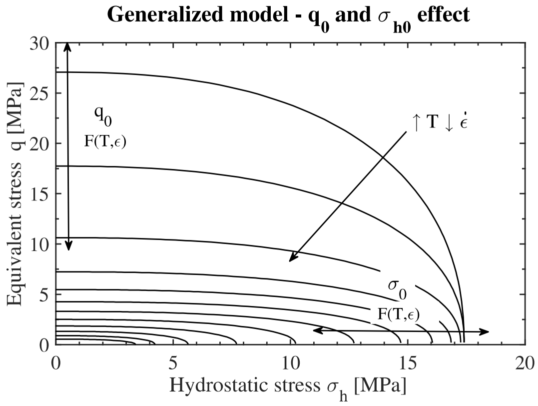


Fig. 5 Effects of q_0 and $\sigma_{h.0}$ parameters on the meridian curve of the proposed model

The model parameter q_0 and $\sigma_{h.0}$ are computed respectively by means of equations (16) and (17). These equations describe the effects of temperature and strain rate variation to the failure surface (see Fig. 5). The meridian of the model tends to converge towards the origin when q_0 and $\sigma_{h.0}$ reduces i.e. at increasing temperature and decreasing strain rate. Both parameters are defined as the product of a reference value, q_v and $\sigma_{h,n}$, with analytical temperature and strain rate function. The analytical temperature and strain rate functions of equations (16) are derived using a probabilistic approach developing a non-linear multi-dimensional algorithm. The developed probabilistic algorithm is developed such as allows the derivation of (i) a model defined over multiple dimensions (in this case the four dimensions mentioned above) (ii) a model with a variable standard deviation and (iii) a model dependent on one or more parameters. More detailed information on the analytical expressions of these functions can be found in (Santarsiero 2015).

3.3. Analytical comparisons with existing models

Equation (14) is now compared with existing models. It is observed that some existing failure criteria can be seen as particular cases of the proposed model with specific values of the model parameters. A first case, for example, is to assume $\beta_q = 1$ and a that tends to zero. With this particular choice of parameters, the model converges to the von Mises criteria. Then, with the same parameters, at Lode's angle equal to 0° and 60°, Tresca is also a particular case of the proposed model (assuming $\sigma_{h.0}$ that tends to infinity). A second case is to assume $\beta_q = 1$ and $\beta_h = 1$. This particular case of the model converges to the Drucker-Prager criterion. At Lode's angle equal to 0 and 60°, the Coulomb-Mohr criterion is also a particular case of the proposed model. In the case of $\beta_q = 2$ and $\beta_h = 1$ it can be observed that the Raghava (Raghava & Caddell 1973) and Balandin (Balandin 1937) models are equivalent to this particular case of the proposed model. Finally, with $\beta_q = 1$ and $\beta_h = 2$ the proposed failure criterion converges to the model given by Gibson in (Gibson et al. 1989) and with $\beta_q = 2$ and $\beta_h = 2$ the proposed failure criterion converges to the model given by Deshpande in (Deshpande & Fleck 2001).

3.4. Hypothesis and analytical limitations of the proposed model

The analytical limitations of the proposed model are listed below:

- The main hypothesis assumed in the analytical definition of the GTM is to be independent of the third deviatoric stress invariant, similar to what is done in von Mises and Drucker-Prager model. However, this hypothesis could be overcome by running the algorithm of the model introducing an analytical term involving the Lode's angle.

- Given the Drucker's stability postulates and the subsequent convexity condition, the following mathematical limitations apply to the model parameters:

$$\beta_h > 1 \quad ; \quad \beta_q > 1 \tag{19}$$

- The proposed model is here used to analyse stress state with positive value of the hydrostatic component of the stress tensor. In case of negative hydrostatic stress, the following mathematical limitation applies:

$$\beta_h \in \mathbb{Z} \tag{20}$$

- In equation (14), the exponents β_q and β_h are described by scalar values. It follows that these parameters are strain rate and temperature independent. Further work could be performed modifying equation (14), introducing strain rate and temperature terms in the exponents.

- Hydrostatic-deviatoric multiplicative terms (such as $q\sigma_h$) are not taken into consideration by the proposed model.

4. Conclusion

This work gives an introduction to the theoretical development of a novel failure a criterion for TSSA and SG connections under varying stress state conditions. The main output of this work is a four-dimensional Generalized Triaxial Model (GTM) that accounts for a generic stress state by a governing equation expressed as a function of the three-dimensional stress tensor. Both deviatoric and hydrostatic energetic components are taken into consideration by means of a non-linear function of the two contributions. The proposed model is defined over four independent dimensions: the equivalent stress, the hydrostatic stress, the strain rate and the temperature. In a three-dimensional stress space, the governing equation of the GTM is represented by a revolution surface, axial-symmetric with respect to the hydrostatic axis. The effects of the model parameters are investigated by a parametrical study. It is shown that the exponents of the governing equation affect the orientation of the surface tangent to the hydrostatic axis and to the deviatoric plane. It is also shown that the meridian of the surface tends to the origin of the three-dimensional stress space for decreasing strain rate and increasing temperature. The proposed model is then analytically compared to existing failure criteria. It is shown that some of the existing models available in the literature are particular cases of the proposed model. Although the GTM was theoretically developed specifically for SG and TSSA, the model can be generically used as failure criterion for many isotropic materials.

Acknowledgments

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